Addition and Subtraction of Decimals

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CHAPTER 1 Addition and Subtraction of Decimals

CHAPTER OUTLINE

1.1 Decimal Place Value

Introduction

The Ice Cream Stand

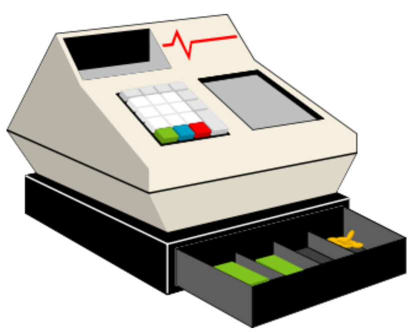
Julie and her friend Jose are working at an ice cream stand for the summer. They are excited because in addition to making some money for the summer, they also get to eat an ice cream cone every day.

On the first day on the job, Julie is handed a cash register drawer that is filled with money. This is the drawer that she can collect money from sales in as well as make change for customers.

Julie needs to count the amount of money in her drawer to be sure that it is accurate. Her boss Mr. Maguire tells her that her drawer should have ***sixty-five dollars and seventy-five cents*** in it.

He hands her a data sheet that she needs to write that money amount in on.

Julie looks at the bills in her drawer and begins to count. She finds 2-20 dollar bills, 2-ten dollar bills, 1-five dollar bill and 2 quarters, 2 dimes and 1 nickel.



Now it is your turn to help.

In this lesson, you will learn all about decimals. One of the most common places that we see decimals is when we are working with money. Your work with decimals and place value will help Julie count her bills and change accurately.

Pay attention so that you can count and write the correct amount of money on Julie's data sheet at the end of the lesson.

What You Will Learn

In this lesson, you will learn how to complete the following tasks:

- Express numbers given in words or hundredths grids using decimal place value.
- Express numbers in expanded form given decimal form.
- Read and write decimals to ten-thousandths place.
- Write combinations of coins and bills as decimal money amounts.

Teaching Time

I. **Express Numbers Given in Words or Hundredths Grids Using Decimal Place Value**

Up until this time in mathematics, we have been working mainly with ***whole numbers***. A whole number represents a whole quantity. There aren't any parts when we work with a whole number.

When we have a part of a whole, we can write it in a couple of different ways. One of the ways that we write it is as a ***decimal***.

A decimal is a part of a whole. Here is an example of a decimal.

Example

4.56

This decimal has parts and wholes in it. Notice that there is a point in the middle of the number. This is called the ***decimal point***. The decimal point helps us to divide the number between wholes and parts. **To the right of the decimal point are the parts of the whole and to the left of the decimal point is the whole number.**

We can have numbers with parts and wholes in them, and we can have numbers that are just decimals.

Example

.43

This decimal has two decimal places. Each digit after the decimal is in a different place. We call these places place values.

When you were working with whole numbers you used place value too, but this is a new place value system that includes decimals.

How can we express a decimal using place value?

To express a decimal using place value we need to use a place value chart. This gives us an idea about the worth of the decimal.

Here is a place value chart.

TABLE 1.1:

Tens	Ones		Tenths	Hundredths	Thousandths	Ten Thousandths
		.				

Notice that if we take the last example and write it in the place value chart above each number is a word. That word gives us the value of that digit according to its place in the chart. This number is forty-three hundredths. The three is the last number, and is in the hundredths place so that lets us know to read the entire number as hundredths.

TABLE 1.2:

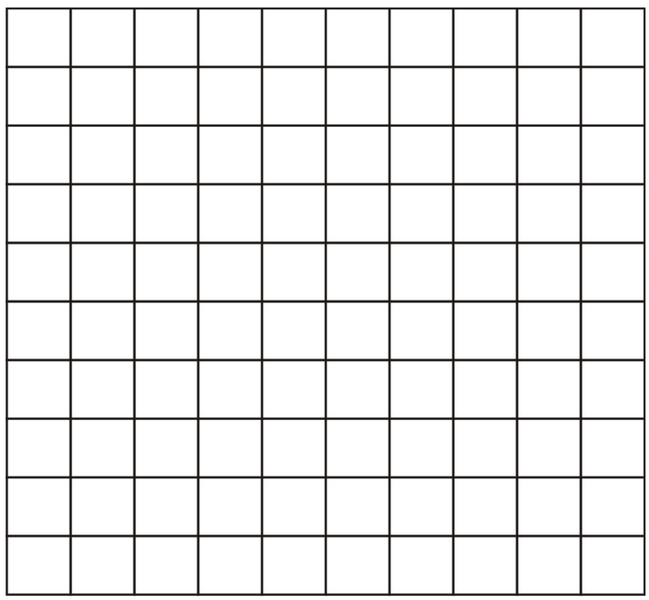
Tens	**Ones**		**Tenths**	**Hundredths**	**Thousandths**	**Ten Thousandths**
		.	**4**	**3**		

Hmmm. Think about that, the word above each digit has a name with a THS in it. The THS lets us know that we are working with a part of a whole.

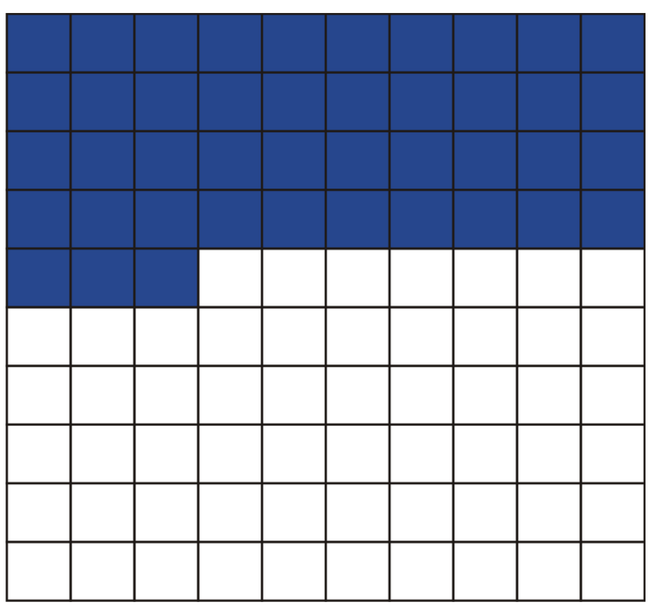
What whole is this decimal a part of?

To better understand what whole the decimal is a part of, we can use a picture. We call these grids or hundreds grids. Notice that the number in the last example was .43 or 43 hundredths. The hundredths lets us know that this is "out of one hundred."

Here is a picture of a hundreds grid.



Now we want to show 43 hundredths of the hundreds grid. To do that, we shade 43 squares. Each square is one part of one hundred.



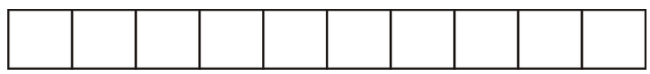
What about tenths?

If you look at a place value chart, you can see that there are other decimal names besides hundredths. We can also have tenths.

Example

.5

Here is a number that is five-tenths. We can create a picture of five-tenths using a grid of ten units.

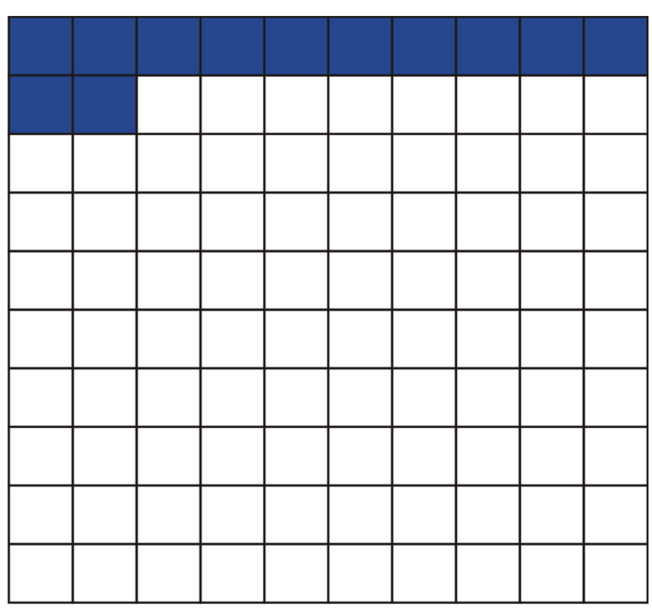


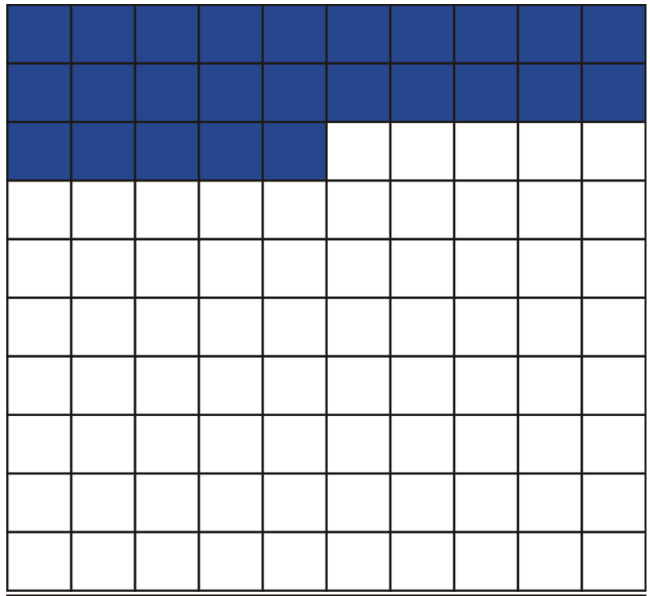
If we want to show .5 in this box, we can see that tenths means 5 out of 10. We shade five boxes of the ten.

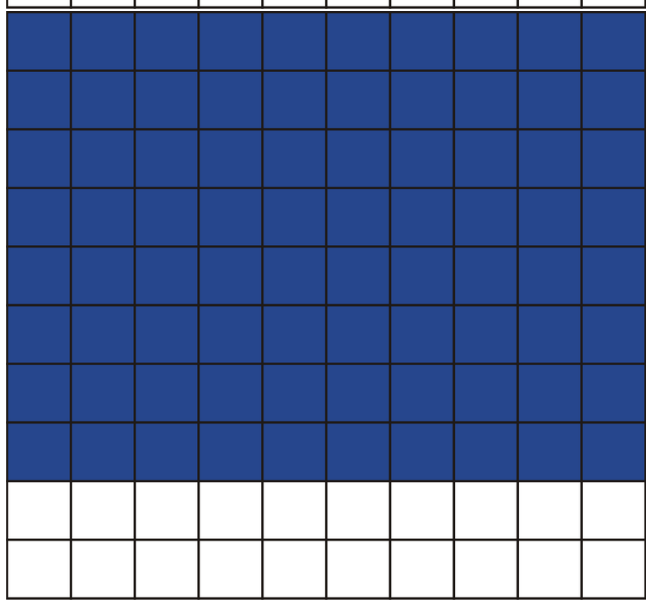
We can make pictures of tenths, hundredths, thousandths and ten-thousands.

Ten-thousandths, whew! Think about how tiny those boxes would be.

Here are a few for you to try. Write each number in words and as a decimal using each grid.

1. 

2. 

3. 

Take a minute to check your work with a peer.

II. **Express Numbers in Expanded Form Given Decimal Form**

We just worked on expressing decimals in words using a place value chart and in pictures using grids with tens and hundreds in them.

We can also stretch out a decimal to really see how much value each digit of the decimal is worth. This is called ***expanded form.***

What is expanded form?

Expanded form is when a number is stretched out. Let's look at a whole number first and then use this information with decimals.

Example

265

If we read this number we can read it as two hundred and sixty-five.

We can break this apart to say that we have two hundreds, six tens and five ones.

HUH??? What does that mean? Let's look at our place value chart to help us make sense of it.

TABLE 1.3:

Hundred	Tens	Ones		Tenths	Hundredths	Thousandths	Ten Thousandths
2	6	5	.				

If you look at the chart you can see how we got those values for each digit. The two is in the hundreds place. The six is in the tens place and the five is in the ones place.

Here it is in expanded form.

2 hundreds + 6 tens + 5 ones

This uses words, how can we write this as a number?

200 + 60 + 5

Think about this, two hundred is easy to understand. Six tens is sixty because six times 10 is sixty. Five ones are just that, five ones.

This is our number in expanded form.

How can we write decimals in expanded form?

We can work on decimals in expanded form in the same way. First, we look at a decimal and put it into a place value chart to learn the value of each digit.

Example

.483

TABLE 1.4:

Hundred	Tens	Ones		Tenths	Hundredths	Thousandths	Ten Thousandths
			.	4	8	3	

Now we can see the value of each digit.

4 = four tenths

8 = eight hundredths

3 = 3 thousandths

We have the values in words, now we need to write them as numbers.

Four tenths = .4

Eight hundredths = .08

Three thousandths = .003

What are the zeros doing in there when they aren't in the original number?

The zeros are needed to help us mark each place. We are writing a number the long way, so we need the zeros to make sure that the digit has the correct value.

If we didn't put the zeros in there, then .8 would be 8 tenths rather than 8 hundredths.

Now, we can write this out in expanded form.

Example

.483

.4 + .08 + .003 = .483

This is our answer in expanded form.

Now it is your turn. Write each number in expanded form.

1. **567**
2. **.345**
3. **.67**

Check your work with a friend to be sure that you are on the right track.

III. **Read and Write Decimals to the Ten-Thousandths Place**

We have been learning all about figuring out the value of different decimals. We have used place value to write them, we have used pictures and we have stretched them out. Now it is time to learn to read and write them directly. Let's start with reading decimals.

How do we read a decimal?

We read a decimal by using the words that show the place value of the last digit of the decimal. That may sound confusing, so let's look at an example.

Example

.45

To help us read this decimal, we can put it into our place value chart.

TABLE 1.5:

Hundred	Tens	Ones		Tenths	Hundredths	Thousandths	Ten Thousandths
			.	4	5		

We read this decimal by using the place value of the last digit to the right of the decimal point.

Normally, we would read this number as **forty-five.**

Because it is a decimal, we read forty-five hundredths. The last digit is a five and it is in the hundredths place.

Can we use place value to write the number too?

Yes we can. We write the number as we normally would.

Example

Forty-five

Next, we add the place value of the last digit to the right of the decimal point.

Forty-five hundredths

Our answer is forty-five hundredths.

We can use this method to read and write any decimal. What about a decimal with more digits?

Example

.5421

First, let's put this number in our place value chart.

TABLE 1.6:

Hundred	Tens	Ones		Tenths	Hundredths	Thousandths	Ten Thousandths
			.	5	4	2	1

First, let's read the number.

We can look at the number without the decimal. It would read:

Five thousand four hundred twenty-one

Next we add the place value of the last digit

Ten thousandth

Five thousand four hundred and twenty-one ten thousandths

This is our answer.

It is also the way we write the number in words too. Notice that is it very important that we add the THS to the end of the place value when working with decimals.

Alright, now you try a few. Write each decimal in words.

1. **.7**
2. **.765**
3. **.2219**

Take a minute to check your work with a peer.

IV. **Write Combinations of Coins and Bills as Decimal Money Amounts**

How can we apply what we have learned in a real world way?

Money is a way that we use decimals every day. Let's think about change.

Coins are cents. If we have 50 pennies, then we have 50 cents. It takes 100 pennies to make one dollar or one whole.

Coins are parts of one dollar. We can represent coins in decimals.

Let's start with pennies.

A penny is one cent or it is one out of 100.

When we have a collection of pennies, we have so many cents out of 100.

Example

5 pennies is 5 cents.

How can we write 5 cents as a decimal?

To do this, we need to think about 5 out of 100.

We can say that 5 cents is 5 hundredths of a dollar since there are 100 pennies in one dollar.

Let's write 5 cents as a decimal using place value.

TABLE 1.7:

Hundred	Tens	Ones		Tenths	Hundredths	Thousandths	Ten Thousandths
			.		5		

The five is in the hundredths box because five cents is five one hundredths of a dollar.

We need to add a zero in the tenths box to fill the gap.

TABLE 1.8:

Hundred	Tens	Ones		Tenths	Hundredths	Thousandths	Ten Thousandths
			.	0	5		

Now we have converted 5 cents to a decimal.

How can we write 75 cents as a decimal?

First, think about what part of a dollar 75 cents is.

Seventy-five cents is seventy-five out of 100.

Now, we can put this into our place value chart.

TABLE 1.9:

Hundred	Tens	Ones		Tenths	Hundredths	Thousandths	Ten Thousandths
			.	7	5		

Now we have written it as a decimal.

What about when we have dollars and cents? Suppose we have twelve dollars and fourteen cents.

A dollar is a whole number amount. Dollars are found to the left of the decimal point.

Cents are parts of a dollar. They are found to the right of the decimal point.

How much money do we have?

There is one ten and the two ones give us twelve dollars.

Then we have some change. One dime and four pennies is equal to fourteen cents.

Here are the numbers:

12 wholes

14 parts

Let's put them in our place value chart.

TABLE 1.10:

Hundred	Tens	Ones		Tenths	Hundredths	Thousandths	Ten Thousandths
	1	2	.	1	4		

There is our money amount.

Our answer is $12.14.

Notice that we added a dollar sign into the answer to let everyone know that we are talking about money.

Real Life Example Completed

The Ice Cream Stand

Now that we know about decimals and money we are ready to help Julie with her ice cream shop dilemma.

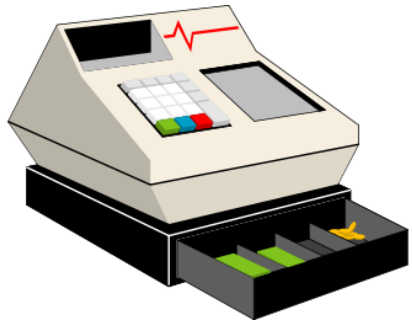
Julie and her friend Jose are working at an ice cream stand for the summer. They are excited because in addition to making some money for the summer, they also get to eat an ice cream cone every day.

On the first day on the job, Julie is handed a cash register drawer that is filled with money. This is the drawer that she can collect money from sales in as well as make change for customers.

Julie needs to count the amount of money in her drawer to be sure that it is accurate. Her boss Mr. Maguire tells her that her drawer should have ***sixty-five dollars and seventy-five cents*** in it.

He hands her a data sheet that she needs to write that money amount in on.

Julie looks at the bills in her drawer and begins to count. She finds 2-20 dollar bills, 2-ten dollar bills, 1-five dollar bill and 2 quarters, 2 dimes and 1 nickel.



First, let's underline all of the important information.

Now, let's count the money she has in the drawer.

1. **How many whole dollars are there?**

There are 2 Twenty Dollar bills = $40 plus 2 Ten Dollar bills = $20 plus 1 Five Dollar bill = $5.

The total then is $40 + $20 + $5 = $65.

2. **How many cents are there?**

There are 2 Quarters at $.25 each = $.50 plus 2 Dimes at $.10 each = $.20 plus 1 Nickel at $.05 = $.05

The total then is $.50 + $.20 + $.05 = $.75

Our next step is to write the wholes and parts in the place value chart. Then we will have this written as a money amount.

TABLE 1.11:

Hundred	Tens	Ones		Tenths	Hundredths	Thousandths	Ten Thousandths
	6	5	.	7	5		

Great work!! Julie has $65.75 in her drawer. That is the correct amount. She is ready to get to work.

Vocabulary

Here are the vocabulary words that can be found in ***italics*** throughout the lesson.

Whole number
: a number that represents a whole quantity

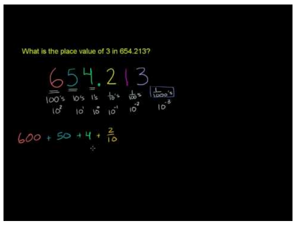
Decimal
: a part of a whole

Decimal point
: the point in a decimal that divides parts and wholes

Expanded form
: writing out a decimal the long way to represent the value of each place value in a number

Technology Integration

This video presents an example of expanded place value.



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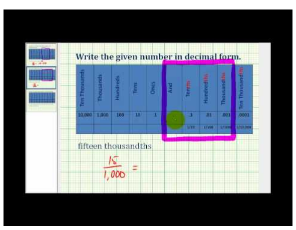
Khan Academy Decimal Place Value



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James Sousa, Write a Number in Decimal Notation from Words

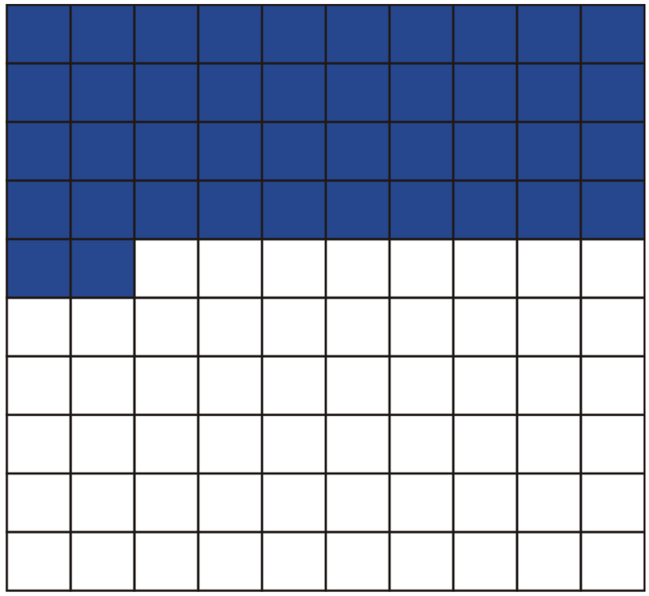
Other Videos:

1. http://www.teachertube.com/viewVideo.php?title=Money_Fractions_and_Decimals&video_id=59116&vpkey=4badb7d45d –This video is a short story and features two students learning about money with fractions and decimals.

Time to Practice

Directions: Look at each hundreds grid and write a decimal to represent the shaded portion of the grid.

1.



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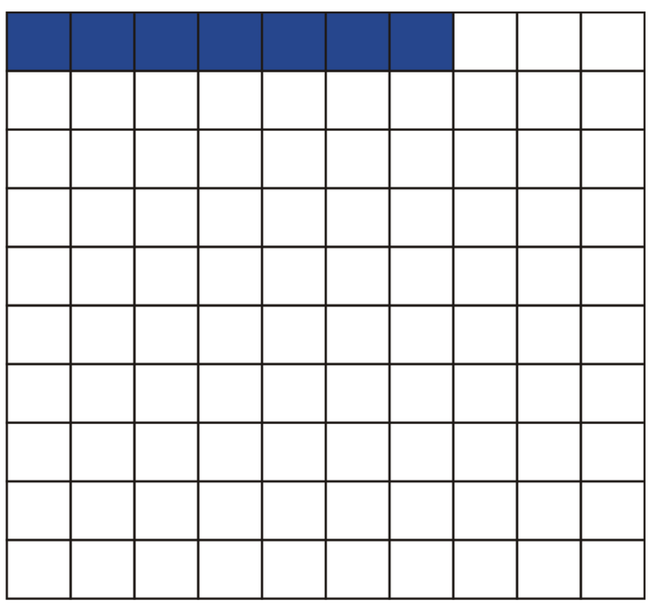

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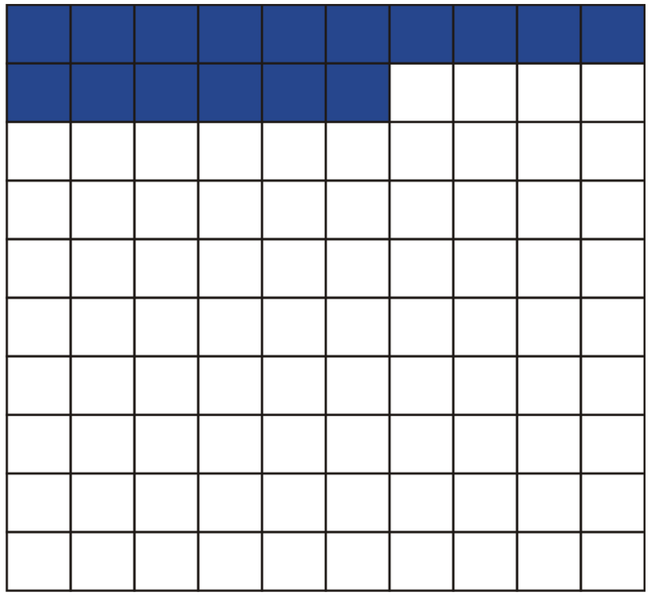

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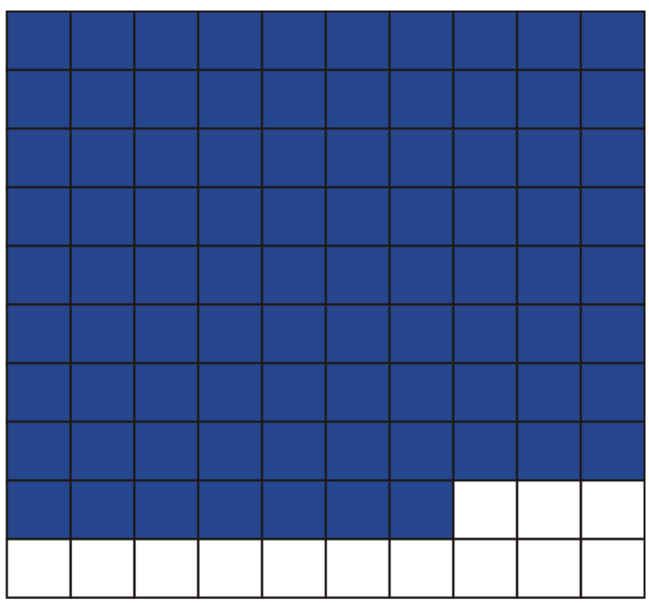

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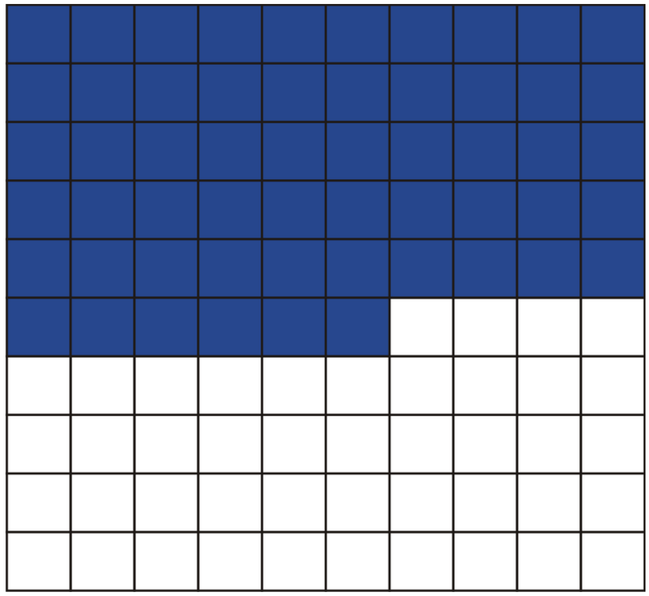


Directions: Write each decimal out in expanded form.

6. .78
7. .345
8. .98
9. .231
10. .986
11. .33
12. .821
13. .4321
14. .8739
15. .9327

Directions: Write out each decimal in words.

16. .4
17. .56
18. .93
19. .8
20. .834
21. .355
22. .15
23. .6
24. .5623
25. .9783

1.2 Measuring Metric Length

Introduction

The Kid's Area

There are a lot of children who visit the ice cream stand each week. Most times they sit with their parents at a large picnic table.

Jose has collected a few small picnic tables to put near each other for a small "kid's area." Mr. Harris loves the idea. Jose gets to work arranging the tables.

Jose has four small picnic tables for his kid's area. He wants to put the tables about 1.5 meters apart. He thinks that this will give the kids plenty of room to not be on top of each other.

He puts out the tables and then gets a ruler and a meter stick. Which tool should Jose use to measure the distance between the two tables?

If he wants the tables to be 1.5 meters apart, how many meter sticks will the distance actually be?

Once Jose gets the tables set up, he wants to design a new placemat for the kids to eat off of. For his placemat, should Jose use a ruler or a meter stick when he measures out the design?

Which makes more sense?

This lesson is all about metric measurement. In the end of the lesson, you will be able to help Jose with his kid's area.

Pay close attention! In the United States we don't have a lot of experience with Metrics. You will need all of the information in this lesson to be successful.

What You Will Learn

In this lesson, you will learn the following skills:

- Identify the equivalence of metric units of length

- Measure lengths using metric units to the nearest decimal place.
- Choose appropriate tools for given decimal metric measurement situations
- Choose appropriate decimal units for given metric measurement situations

Teaching Time

I. **Identify Equivalence of Metric Units of Length**

This lesson focuses on metric units of measurements. In the United States, we use the customary system of measurement more than we use the ***metric system*** of measurement. However, if you travel to another country or complete work in science class, you will need to know metrics.

What are the metric units for measuring length?

When measuring ***length***, we are measuring how long something is, or you could say we are measuring from one end to the other end. That is the length of the item.

Here are the common metric units of length from the smallest unit to the largest unit.

Millimeter

Centimeter

Meter

Kilometer

A ***millimeter*** is the smallest unit. Millimeters are most useful when measuring really tiny things. You can find millimeters on some rulers.

A ***centimeter*** is the next smallest unit. Centimeters can also be found on a ruler.

A ***meter*** is a little more than 3 feet. A meter is a unit that would be very helpful to a carpenter or to someone working in construction.

A ***kilometer*** is used to measure longer distances. You often hear the word kilometer mentioned when talking about a road race that is 5k (or 5 kilometers) long.

How can we convert metric units of length?

When working with the customary units of length: inches, feet, etc., we know that we can convert them from one to another to change the units we are working with. For example, if you have 24 inches, it might make more sense to say that we have 2 feet.

We can do the same thing when working with metric units.

Here is a chart to help us with the conversions.

1 km	1000 m
1 m	100 cm
1 cm	10 mm

Now that you know the conversions, we can change one unit to another unit. Let's look at an example.

Example

5 km = ____ m

Here we are converting kilometers to meters.

How can we convert larger units to smaller units?

We can convert larger units to smaller units by multiplying.

There are 1000 meters in one kilometer.

Example

$$5\ km = \_\_\_\_\_\ m$$
$$5 \times 1000 = 5000\ m$$

Our answer is 5000 m.

Example

600 cm = ______ m

Here we are converting smaller units to larger units.

How can we convert smaller units to larger units?

We can convert smaller units to larger units by dividing.

There are 100 cm in one meter.

Example

$600 \div 100 = 6$

Our answer is 6.

Now it's time for you to try some. Complete the following conversions.

1. **2000 mm = ______ cm**
2. **3 km = ______ m**
3. **4000 cm = ______ m**

Check your answers with a neighbor. Be sure that you both have completed the conversions correctly.

II. **Measure Lengths Using Metric Units to the Nearest Decimal Place**

Sometimes when we convert metric units we don't have a whole number answer. In the last section, all of the examples ended with whole numbers.

Example

2000 mm = 200 cm

These are both whole numbers.

What happens when we convert smaller units to larger units and they don't end up as a whole number?

When this happens, we end up with an answer that is a decimal. If we remember our rules for working with decimals and place value, we can be very successful at converting these small units of measurement to larger units.

Example

1 mm = ______ cm

Here we are converting a smaller unit to a larger unit, because of this we know that we are going to divide.

There are 10 mm in one centimeter, so we are going to divide 1 by 10.

Think about this, we are dividing 1 whole into 10 parts-our answer is definitely going to be a decimal.

$1 \div 10 = .1$ (one tenth)

Our answer is that 1 mm = .1 cm.

We can also round our answer to the nearest tenth.

What if we had a problem where we wanted to convert 1.5 mm to cm?

Example

1.5 mm = ______ cm

Once again, we are going to be dividing by 10.

When we divide by 10 in this example we end up with an answer of .15

1.5 mm = .15 cm

We can round this answer to the nearest tenth.

.15 rounds to .2

We can say that .2 is the closest tenth of a cm to 1.5 mm.

Just as we were able to round whole numbers, we can round decimal measurements too.

Let's look at another example where we will get a decimal answer.

Example

1 m = ______ km

Here we are converting a smaller unit to a larger unit.

There are 1000 meters in one kilometer. We divide by 1000.

$\mathbf{1 \div 1000 = .001}$

Here our answer is one-thousandth of a kilometer.

Now it is time for you to try a few.

1. **2 m = ______ km**
2. **8 mm = ______ cm**
3. **4 cm = ______ m**

III. <u>**Choose Appropriate Tools Given Decimal Metric Measurement Situations**</u>

Now that you have learned all about converting different measurements, it is time to think about which tools to use to measure different things.

We know some metric units for measuring length are millimeters, centimeters, meters and kilometers.

Millimeters and centimeters are found on a ruler.

There is a meter stick that measures 1 meter.

A metric tape measure can be used to measure multiple meters.

If you wanted to measure long distances, you could use a kilometer odometer, like in a car, to measure distance.

What tool should we use when?

A tool is designed to make measuring simpler. If we have a difficult time choosing an appropriate tool, or choose a tool that isn't the best choice, it can make measuring very challenging.

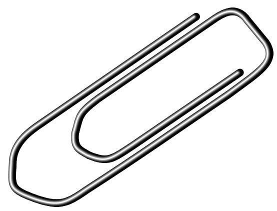
Let's think about tools and when we should use them depending on what and/or where we are measuring.

Here are some general suggestions:

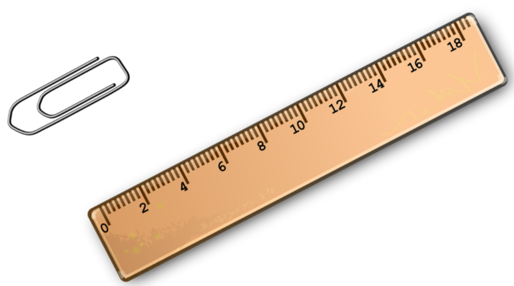
If the object is very tiny, use a ruler for millimeters. If the object is less than 30 cm use a ruler for centimeters. If the object is between 30 cm and 5 or so meters use a meter stick. If the object is greater than a few meters, use a metric tape measure. If the object is a long distance, for instance across town, use a kilometer odometer.

Example

What would we use to measure the following object?



This object is a paperclip. It is definitely smaller than the length of a ruler, so we can use a ruler to measure it.

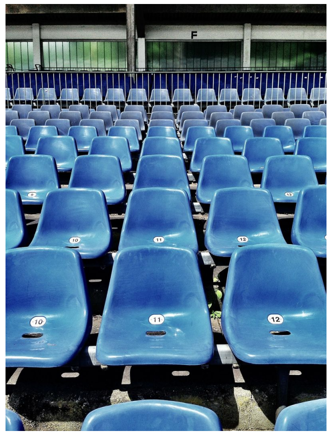


Example

What about measuring a road race?

A road race is usually a significant distance, so we are going to use a kilometer odometer to measure it.

Now it is time for you to choose an appropriate tool.

1. The width of a table
2. 
3. An ant

Take a minute to check your work with a peer. Discuss any differences in your answers.

IV. **Choose Appropriate Decimal Units for Given Metric Measurement Situations**

Now that we know about using the appropriate tool, we also need to choose the best unit to measure different things.

The common metric units of length are ***millimeter, centimeter, meter and kilometer.***

When is the best time to use each measurement?

You can think about this logically. Let's start with millimeters.

A millimeter is the smallest unit. There are 10 mm in one centimeter, if an object is smaller than one centimeter, then you would use millimeters.

Who would use millimeters? A scientist measuring something under a magnifying glass might use millimeters to represent a tiny specimen.

A centimeter is the next smallest unit. We can use a ruler to measure things in centimeters. If an object is the length of a ruler or smaller, then it makes sense to use centimeters to measure.

Meters are used to measure everything between the length of a ruler and the distance between two cities or places.

Most household objects such as tables, rooms, window frames, television screens, etc would be measured in meters.

Kilometers are used to measure distances. If we are looking to figure out the length of a road, the distance between two locations, etc, we would use kilometers.

Think about each example, which is the best unit to measure the objects listed below?

1. **The height of a picture on the wall**
2. **A caterpillar**

3. **The width of a penny**

Explain your answers to a neighbor. Be sure to justify why you chose each unit of measurement.

Real Life Example Completed

The Kid's Area

Now that you have worked with the Metric System, let's go back and look at Jose's work with the kid's area.

Here is the problem once again.

There are a lot of children who visit the ice cream stand each week. Most times they sit with their parents at a large picnic table.

Jose has collected a few small picnic tables to put near each other for a small "kid's area." Mr. Harris loves the idea. Jose gets to work arranging the tables.

Jose has four small picnic tables for his kid's area. He wants to put the tables about 1.5 meters apart. He thinks that this will give the kids plenty of room to not be on top of each other.

He puts out the tables and then gets a ruler and a meter stick. Which tool should Jose use to measure the distance between the two tables?

If he wants the tables to be 1.5 meters apart, how many meter sticks will the distance actually be?

Once Jose gets the tables set up, he wants to design a new placemat for the kids to eat off of. For his placemat, should Jose use a ruler or a meter stick when he measures out the design?

Which makes more sense?

First, let's underline the important questions and information in this problem.

Now let's look at the first question. Jose wants to measure a distance that is much longer than a ruler. He could use a ruler, but think about how many centimeters are in one meter. If Jose is wishing to make his work the simplest that it can be, then he should use the meter stick.

For 1.5 meters, Jose would have to measure out 150 centimeters.

If Jose uses the meter stick, then he would need to measure one and one-half lengths of the meter stick to have the accurate measurement between the tables.

For the placemat design, Jose is going to be working with a much smaller area. He can use a ruler for this design since most placemats are about the size of a piece of paper. Jose will be able to work well with his ruler while a meter stick would be very difficult to work with.

Vocabulary

Here are the vocabulary words that can be found in this unit.

Metric System
a system of measurement more commonly used outside of the United States

Length
the measurement of a object or distance from one end to the other

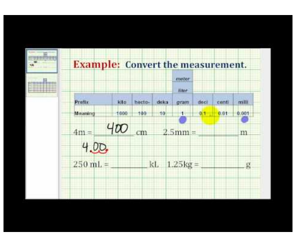
Millimeter
the smallest common metric unit of measuring length, found on a ruler

Centimeter
a small unit of measuring length, found on a ruler

Meter
approximately 3 feet, measured using a meter stick

Kilometer
a measurement used to measure longer distances, the largest common metric unit of measuring length

Technology Integration



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James Sousa, Converting Between Metric Units

Other Videos:

1. http://www.mathplayground.com/howto_Metric.html –This video expands on the basic information of the metric system. It also begins working with metric conversions.
2. http://www.teachertube.com/viewVideo.php?video_id=8896 –The Metric System song to "Arms Wide Open" by Creed this is sung by two science teachers.

Time to Practice

Directions: Complete the following metric conversions.

1. 6 km = ______ m
2. 5 m = ______ cm
3. 100 cm = ______ m
4. 400 cm = ______ m
5. 9 km = ______ m
6. 2000 m = ______ km
7. 20 mm = ______ cm
8. 8 cm = ______ mm
9. 900 cm = ______ m
10. 12 m = ______ cm

Directions: Write each decimal conversion. Round to the nearest hundredth when necessary

11. 1 mm = ______ cm
12. 5 mm = ______ cm
13. 8 cm = ______ m
14. 9 cm = ______ m
15. 12 m = ______ km
16. 8 m = ______ km
17. 22 mm = ______ cm
18. 225 mm = ______ cm
19. 543 mm = ______ cm
20. 987 mm = ______ cm

Directions: Choose the best tool to measure each item. Use ruler, meter stick, metric tape or kilometric odometer.

21. A paperclip
22. The width of a dime
23. A tall floor lamp
24. The width of a room
25. A road race from start to finish

Directions: Choose the best metric unit for each measurement situation.

26. The length of a small table

27. A book
28. A cell phone
29. The length of a room
30. The distance from Boston to Cincinnati

1.3 Ordering Decimals

Introduction

Sizing Up Ice Cream Cones

So far Julie is really enjoying working at the ice cream stand. She loves talking with the people and the ice cream snacks are definitely a benefit.

However, she is very confused about the size of the ice cream cones.

Mr. Harris, the stand owner, used to be a math teacher so he loves to have fun with the customers. Because of this, the stand serves cones in different measurement units. It is famous for its mathematical ice cream cones.

This has been very frustrating for Julie.

Yesterday, a customer wanted to know whether a Kiddie Cone 1 was smaller or larger than a Kiddie Cone 2. One is in centimeters and one is in millimeters.

A second customer came in and wanted to know if the Small cone was larger than a Big Kid cone. Again, Julie didn't know what to say.

Here is the chart of cone sizes.

Kiddie cone 1 = 80 mm

Kiddie cone 2 = 6 cm

Big Kid cone = 2.25 inches

Small cone = 2.5 inches

Julie went to see Mr. Harris for help, but he just chuckled.

"It is time to brush up on your measurement and decimals my dear," he said smiling.

Julie is puzzled and frustrated.

Would you know what to say to the customers?

In this lesson, you will learn all about comparing. This lesson will teach you how to figure out which decimal or measurement is greater and which is smaller.

Hopefully, we will be able to help Julie at the end of the lesson.

What You Will Learn

In this lesson you will learn the following skills:

- Comparing Metric lengths
- Comparing decimals
- Ordering decimals
- Describing real-world portion or measurement situations by comparing and ordering decimals.

Teaching Time

I. **Comparing Metric Length**

In our last lesson we learned how to convert metric lengths. We learned that there are 10 millimeters in one centimeter and that we can change millimeters to centimeters by dividing. We also learned that we can change centimeters to millimeters by multiplying.

We can call these measurements ***equivalents.***

The word equivalent means equals. When we know which measurement is equal to another measurement, then we can tell what is equal to what.

Here is a measurement chart of equivalents.

$$\begin{aligned}1\ cm &= 10\ mm\\1\ m &= 100\ cm\\1\ m &= 1000\ mm\\1\ km &= 1000\ m\end{aligned}$$

Let's say that we wanted to compare two different units to figure out which is greater and which is less. We could use the chart to help us.

Here is an example.

Example

5 cm ______ 70 mm

1. **The first thing that we need to do is to convert the measurements so that the unit of measurement is the same.**

Here we have cm and mm. We need to have either all mm or all cm. It doesn't matter which one we choose as long as it is the same unit. Let's use cm.

$$70\ mm = \_\_\_\_\_\ cm$$
$$70 \div 10 = 7$$

Our answer is 7 cm.

2. **Let's rewrite the problem.**

5 cm ______ 7 cm

3. **Use greater than >, less than <, or equal to = to compare the measurements.**

Example

5 cm <7 cm

So 5 cm <70 mm

Take a minute to write down a few notes on these steps.

We can easily compare any two measurements once we have converted them to the same unit of measure.

Let's look at another example

Example

7000 m ______ 8 km

Here we have two different units of measurement. We have meters and kilometers.

Our first step is to convert both to the same unit. Let's convert to meters this time.

8 km $= 8 \times 1000 =$ 8000 m

Now we can compare.

7000 m <8000 m

Our answer is 7000 m <8 km.

Here are a few for you to try on your own. Use <, >, or = to compare.

1. **7 m ______ 7000 mm**
2. **3 km ______ 3300 m**
3. **1000 mm ______ 20 cm**

Stop and check your work with a peer.

II. **Compare Decimals**

We just finished comparing metric lengths. All of the work that we did was with whole units of measurement. We compared which ones were greater than, less than or equal to. What if we had been working with decimals?

How can we compare decimals?

When we ***compare decimals***, we are trying to figure out which part of a whole is greater. To do this, we need to think about the number one.

1 is a whole. All decimals are part of one.

The closer a decimal is to one, the larger the decimal is.

How can we figure out how close a decimal is to one?

This is a bit tricky, but if we look at the numbers and use place value we can figure it out.

Let's look at an example.

Example

.45 ______ .67

Here we have two decimals that both have the same number of digits in them. It is easy to compare decimals that have the same number of digits in them.

Now we can look at the numbers without the decimal point. Is 45 or 67 greater?

67 is greater. We can say that sixty-seven hundredths is closer to one than forty-five hundredths.

This makes sense logically if you think about it.

Our answer is .45 <.67.

Steps for Comparing Decimals

1. **If the decimals you are comparing have the same number of digits in them, think about the value of the number without the decimal point.**
2. **The larger the number, the closer it is to one.**

What do we do if the decimals we are comparing don't have the same number of digits?

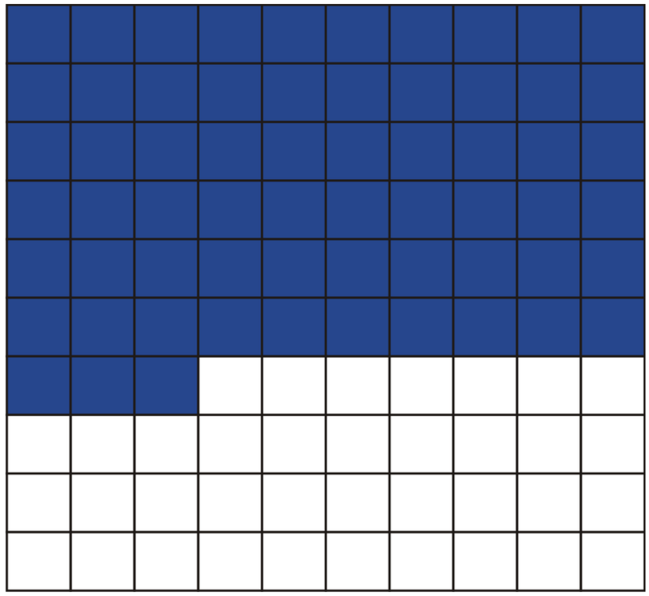
Example

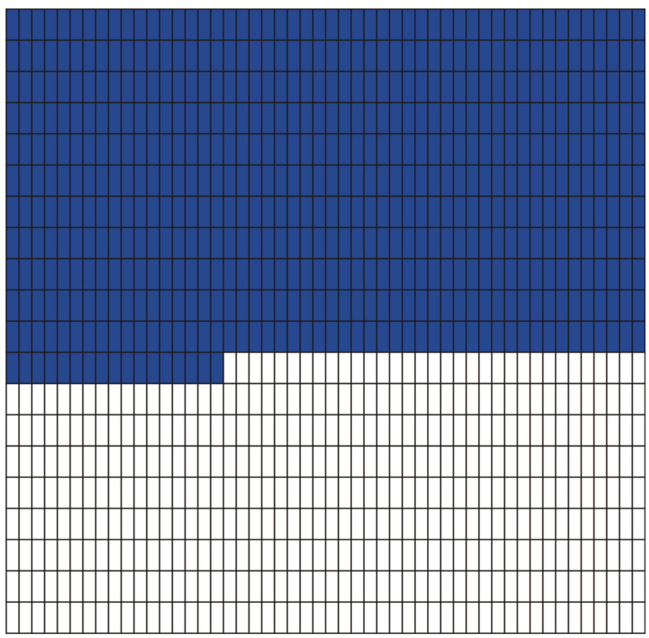
.567 ______ .64

Wow, this one can be confusing. Five hundred and sixty-seven thousandths *seems* greater. After all it is thousandths. The tricky part is that thousandths are smaller than hundredths.

Is this true?

To test this statement let's look at a hundreds grid and a thousands grid.





Now it is easier to compare. You can see that .64 is larger than .567.

How can we compare without using a grid?

Sometimes, we don't have a grid to look at, what then?

We can add zeros to make sure that digit numbers are equal. Then we can compare.

Let's do that with the example we have been working on.

Example

.567 _______ .640

That made comparing very simple. 640 is larger than 567.

Our answer is that .567 <.640.

What about a decimal and a whole number?

Sometimes, a decimal will have a whole number with it. If the whole number is the same, we just use the decimal part to compare.

Example

3.4 ______ 3.56

First, we add in our zeros.

3.40 ______ 3.56

The whole number, 3 is the same, so we can look at the decimal.

40 is less than 56 so we can use our symbols to compare.

Our answer is 3.4 <3.56.

Can you work these out on your own? Compare the following decimals using <, >, or =.

1. **.0987 ______ .987**
2. **.453 ______ .045**
3. **.67 ______ .6700**

How did you do? Take a minute to check your answers with a neighbor.

III. **Order Decimals**

Now that we know how to compare decimals, we can ***order*** them. Ordering means that we list a series of decimals according to size. We can write them from least to greatest or greatest to least.

How can we order decimals?

Ordering decimals involves comparing more than one decimal at a time. We need to compare them so that we can list them.

Here is an example for us to work with.

Example

.45, .32, .76

To write these decimals in order **from least to greatest**, we can start by comparing them.

The greater a decimal is the closer it is to one whole.

The smaller a decimal is the further it is from one whole.

Just like when we compared decimals previously, the first thing we need to look at is the digit number in each decimal. These each have two digits in them, so we can compare them right away.

Next, we can look at each number without the decimal and write them in order from the smallest to the greatest.

Example

.32, .45, .76

32 is smaller than 45, 45 is greater than 32 but smaller than 76, 76 is the largest number

Our answer is .32, .45, .76

What if we have decimals with different numbers of digits in them?

Example

Write these in order from greatest to least:

.45, .678, .23

Here we have two decimals with two digits and one decimal with three. We are going to need to create the same number of digits in all three decimals. **We can do this by adding zeros.**

Example

.450, .678, .230

Now we can write them in order from greatest to least.

Our answer is .23, .45, .678.

Now it is time for you to apply what you have learned. Write each series in order from least to greatest.

1. **.6, .76, .12, .345**
2. **.34, .222, .6754, .5, .9**
3. **.78, .890, .234, .1234**

Take a minute to check your work with a peer.

Real Life Example Completed

Sizing Up Ice Cream Cones

Okay, now you have learned all about comparing measurement and decimals, so we can get back to Julie and the ice cream cones.

Let's take another look at the problem first.

So far Julie is really enjoying working at the ice cream stand. She loves talking with the people and the ice cream snacks are definitely a benefit.

However, she is very confused about the size of the ice cream cones.

Mr. Harris, the stand owner, used to be a math teacher so he loves to have fun with the customers. Because of this, the stand serves cones in different measurement units. It is famous for its mathematical ice cream cones.

This has been very frustrating for Julie.

Yesterday, a customer wanted to know whether a Kiddie Cone 1 was smaller or larger than a Kiddie Cone 2. One is in centimeters and one is in millimeters.

A second customer came in and wanted to know if the Small cone was larger than a Big Kid cone. Again, Julie didn't know what to say.

Here is the chart of cone sizes.

Kiddie Cone 1 = 80 mm

Kiddie Cone 2 = 6 cm

Big Kid cone = 2.25 inches

Small cone = 2.5 inches

Julie went to see Mr. Harris for help, but he just chuckled.

"It is time to brush up on your measurement and decimals my dear," he said smiling.

First, let's underline all of the important information.

Next, we can see that there are two customers who had questions.

Let's look at the first customer's question.

The first customer is comparing Kiddie Cone 1 with Kiddie Cone 2. Let's look at the measurements for each of these cones.

Kiddie Cone 1 = 80 mm

Kiddie Cone 2 = 6 cm

We need to convert the units both to millimeters or both to centimeters.

Let's use cm. We go from a smaller unit to a larger unit so we divide. There are 10 mm in 1 centimeters therefore we divide by 10.

$80 \div 10 = 8$

Kiddie Cone 1 = 8 cm

Kiddie Cone 2 = 6 cm

$8 > 6$

Kiddie Cone 1 is greater than Kiddie Cone 2.

The second customer wanted to know whether the Big Kid Cone was larger or smaller than the Small cone.

These cones have measurements in decimals, so we need to compare the decimals.

Big Kid cone = 2.25

Small cone = 2.5

The whole number is the same, 2, so we can compare the decimal parts.

.25 and .50

.25 <.50

2.25 <2.5

The Big Kid cone is smaller than the Small cone.

Julie is relieved. She now understands comparing decimals and measurement. Next time, she will be ready to answer any of the customer's questions.

Vocabulary

Here are the vocabulary words that can be found in this section.

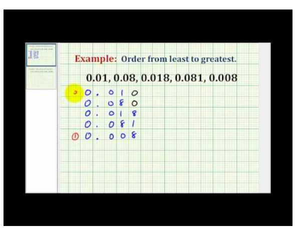
Equivalent
means equal

Comparing
using greater than, less than or equal to so that we can compare numbers

Decimals
a part of a whole represented by a number to the right of a decimal point

Order
writing numbers from least to greatest or greatest to least

Technology Integration



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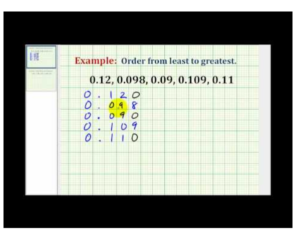
James Sousa, Example of Ordering Decimals from Least to Greatest



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URL: http://www.ck12.org/flx/render/embeddedobject/5320

James Sousa, A Second Example of Ordering Decimals from Least to Greatest

Other Videos:

1. http://www.linkslearning.org/Kids/1_Math/2_Illustrated_Lessons/3_Place_Value/index.html –A GREAT video that starts with whole numbers and moves through to decimals. It really provides a clear understanding of the concepts.

Time to Practice

Directions: Compare metric lengths using <, >, or =

1. 6 cm ______ 60 mm

2. 8 cm ______ 90 mm

3. 10 mm ______ 4 cm

4. 40 mm ______ 6 cm

5. 5 km ______ 4000 m

6. 7 km ______ 7500 m

7. 11 m ______ 1200 cm

8. 9 km ______ 9000 m

9. 100 mm ______ 750 cm

10. 18 km ______ 1500 m

Directions: Compare the following decimals using <, >, or =

11. .4 ______ .2

12. .67 ______ .75

13. .90 ______ .9

14. .234 ______ .54

15. .123 ______ .87

16. .954 ______ .876

17. .32 ______ .032

18. .8310 ______ .0009

19. .9876 ______ .0129

20. .8761 ______ .9992

Directions: Order the following decimals from least to greatest.

21. .8, .9. .2,. 4

22. .02, .03, .07, .05, .04

23. .34, .21, .05, .55

24. .07, .7, .007, .0007

25. .87, 1.0, .43, .032, .5

26. .067, .055, .023, .011, .042

27. .55, .22, .022, .033, .055

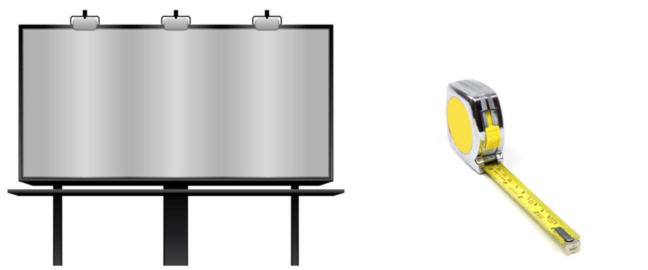
28. .327, .222, .0222, .321, .4

29. .65, .6, .67, .678, .69
30. .45, .045, 4.5, .0045, .00045

1.4 Rounding Decimals

Introduction

The New Ice Cream Sign



Mr. Harris has given Jose the task of creating a new sign for "Add It Up Ice Cream". The paint on the old sign is chipped and peeling, so Mr. Harris is hoping for a beautiful new sign to attract business.

Jose loves to paint and design things so he is the right person for the job. Jose is excited. He takes down the old sign and begins thinking about how he is going to design it.

Here is some of the information that Jose has to work with.

- The original sign is 4.25' $\times$ 2.5'
- The letters on the original sign are 1.67' high

While Jose is working on his drawing, Mr. Harris walks up behind him.

"Jose, I think we should work with a new sign board too. Please round the length of the sign to the nearest half foot and the width to the nearest whole foot. Also, please make the letters a bit larger than the original. Maybe round up to the nearest foot on those too," Mr. Harris says to Jose with a twinkle in his eye.

Jose smiles at Mr. Harris and then shrugs when Mr. Harris walks away.

Jose will need to remember how to round decimals for this plan to work.

In this lesson, you will need to learn how to round decimals to help Jose.

Pay close attention, we will be using what we learn in this lesson to help Jose with his new sign.

What You Will Learn

In this lesson you will learn the following skills:

- Round decimals using a number line.
- Round decimals given place value.
- Round very small decimal fractions to the leading digit
- Round very large numbers to decimal representations of thousands, millions, etc.

Teaching Time

Think about Jose. He is using decimals to design a new sign. His problem is an example of how decimals can show up in real life. Not all measurements are whole number measurements. Often we have measurements that are written in parts, decimals.

Sometimes, it is easier to ***round*** a decimal to the nearest whole or large part.

In this lesson, we are going to be learning how to round decimals.

I. **Rounding Decimals Using a Number Line**

Let's think back for a minute to rounding whole numbers. When we were rounding whole numbers, we could round a number to any place value that we wanted to. We could round to tens, hundreds, thousands, etc.

To do this, we followed a few simple rules.

1. Look at the digit to the right of the place value you are rounding.
2. If the digit to the right is a five or greater, you round up.
3. If the digit to the right is less than 5, you round down.

Let's look at an example to help us remember.

Example

Round the number 46 to the nearest ten

The four is in the tens place, that is the place we are rounding.

The six is in the ones place, that is the digit we look at.

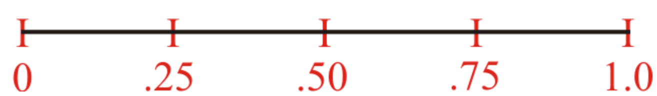
Since 6 is a five or greater, we round up.

46 becomes 50.

Our answer is 50.

There are a couple of different ways that we can round decimals.

First, let's look at rounding them using a number line.



Here we have a number line. You can see that it starts with zero and ends with one. This number line has been divided up into quarters.

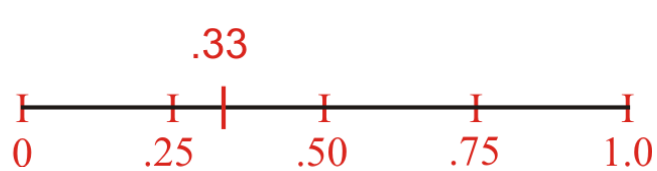
It goes from 0 to .25 to .50 to .75 to 1.0.

Let's look at an example that we were going to round to the nearest quarter.

Example

.33

Here we have .33. The first thing that we want to do is to graph it on a number line.



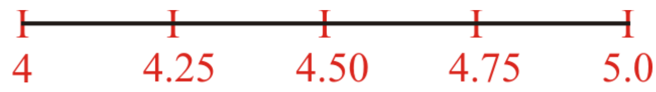
We want to round to the nearest quarter. This number line gives us a terrific visual to do this.

Which quarter is .33 closest to?

It is closest to .25.

Our answer is .25.

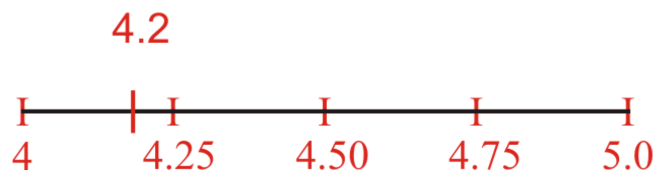
We can also round decimals to the nearest whole using a number line.



Example

Round 4.2 to the nearest whole number.

Here we can use our number line to show us which whole number 4.2 is closest too.



Wow! It is great to be able to see this so clearly.

Is 4.2 closer to 4.0 or 5.0 on the number line?

It is closer to 4.0.

Our answer is 4.0.

II. **Rounding Decimals to A Given Place Value**

We can also use place value to help us in rounding numbers.

Once again, we are going to follow the same rules that we did when rounding whole numbers, except this time we will be rounding to the nearest whole or tens, hundreds, thousands, etc.

Let's look at an example.

Example

Round .345 to the nearest tenth

To help us with this, let's put the number in our place value chart.

TABLE 1.12:

Tens	**Ones**		**Tenths**	**Hundredths**	**Thousandths**	**Ten Thousandths**
		.	**3**	**4**	**5**	

Now we are rounding to the nearest tenth.

3 is in the tenths place.

4 is the digit to the right of the place we are rounding.

It is less than 5, so we leave the 3 alone.

Our answer is .3.

Notice that we don't include the other digits because we are rounding to tenths. We could have put zeros in there,

but it isn't necessary.

Example

Round .567 to the nearest hundredth

To help us with this, let's use our place value chart again.

TABLE 1.13:

Tens	Ones		Tenths	Hundredths	Thousandths	Ten Thousandths
		.	**5**	**6**	**7**	

Now we are rounding to the nearest hundredth.

The 6 is in the hundredths place.

The 7 is the digit to the right of the hundredths place.

Since a 7 is 5 or greater, we round up to the next digit.

6 becomes 7.

Our answer is .57.

Notice in this case that the five is included. Because it is to the left of the place we are rounding, it remains part of the number.

Now it's time for you to practice, round each number using place value.

1. **Round to the nearest tenth, .892**
2. **Round to the nearest hundredth, .632**
3. **Round to the nearest thousandths, .1238**

Take a minute to go over your work with a neighbor.

III. **Round Very Small Decimal Fractions to the Leading Digit**

We know that a decimal is a part of a whole. We also know that some decimals are smaller than others. If we have a decimal that is 5 tenths of a whole, this is a larger decimal than 5 hundredths of a whole. Let's look at those two decimals.

Example

.5 ______ .05

If we were going to compare these two decimals, we would add a zero to the first decimal so that it has the same number of digits as the second.

.50 >.05

We can see that the five tenths is greater than five hundredths.

This example can help us to determine very small decimals.

A decimal is a very *small decimal* depending on the number of places represented after the decimal point. The more decimal places, the smaller the decimal is.

Example

.000056787

Wow! That is a lot of digits. Because this decimal has so many digits, we can say that it is a very tiny decimal.

We can round tiny decimals like this one too. We use something called the *leading digit* to round a very small decimal.

The leading digit is the first digit of the decimal that is represented by a number not zero.

In this example, the leading digit is a five.

Example

.000056787

To round this decimal, we use the leading decimal and add in the rounding rules that we have already learned.

The digit to the right of the five is a six.

Six is greater than 5, so we round up.

Our answer is .00006.

Notice that we include the zeros to the left of the leading digit, but we don't need to include any of the digits after the leading digit. That is because we rounded that digit so we only need to include the rounded part of the number.

We can find very small decimals in real life too. Look at this example.

Example

On August 5, 2007, the Japanese yen was worth .008467 compared to the US dollar.

Let's say we wanted to round the worth of the yen to the leading digit.

First, let's find the leading digit. The first digit represented by a number not a zero is 8.

Now we apply our rounding rules.

The digit to the right of the 8 is a 4. So the 8 remains the same.

Our answer is .008

It is your turn to apply this information, round each small decimal by using the leading digit.

1. **.0004567**
2. **.0000178923**
3. **.00090034**

Take a minute to check your work with a peer. Did you remember which value was the leading digit?

IV. Rounding Very Large Numbers to Decimal Representations of Thousands, Millions, etc.

We just finished rounding some very tiny numbers, but what about really large numbers? Can we use rounding to help us to examine some really large numbers?

Let's think about this.

Every time a new movie comes out a company keeps track of the total of the movie sales. If you go to www.the-numbers.com/movies/records you can see some of these numbers.

Here are the sales totals for the three top movies according to movie sales.

1. Star Wars IV - $460,998,007
2. Avatar - $558,179,737
3. Titanic - $600,788,188

Wow! Those are some big numbers!

Here is where rounding can be very helpful.

We can round each of these numbers to the nearest hundred million.

First, let's find the hundred millions place.

1. **Star Wars IV** - **$460,998,007**
2. **Avatar** - **$558,179,737**
3. **Titanic** - **$600,788,188**

We want to round to the nearest hundred million. We do this by looking at the number to the right of the place that we are rounding.

Let's look at each movie individually.

1. Star Wars IV - The number after the 4 is a 6, so we round up to a 5. The rest of the numbers are zeros.

500,000,000

2. Avatar - The number after the 5 is a 5, so we round up to 6. The rest of the numbers are zeros.

600,000,000

3. Titanic - The number after the 6 is a zero. So the 6 stays the same and the rest of the numbers are zeros.

600,000,000

If we want to compare these numbers now we can see that Avatar and Titanic had the highest sales and Star Wars IV had the least sales.

Sometimes we can get confused reading numbers with so many digits in them. Rounding the numbers helps us to keep it all straight.

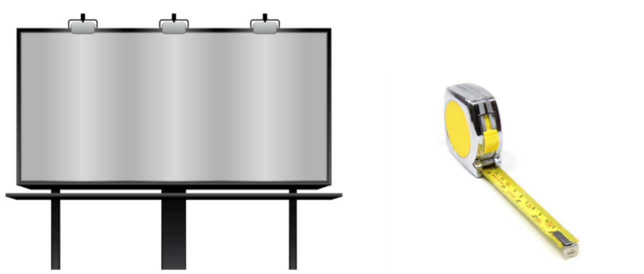
Here are a few for you to try. Round each to the correct place.

1. **Round the nearest million, 5,689,432.**
2. **Round to the nearest hundred thousand, 789,345**
3. **Round to the nearest billion, 3,456,234,123**

Take a minute to check your work with a peer.

Real Life Example Completed

The New Ice Cream Stand



Now that you have had a chance to learn about rounding decimals, you are ready to help Jose with his dilemma.

Let's look at the problem once again.

Mr. Harris has given Jose the task of creating a new sign for "Add It Up Ice Cream". The paint on the old sign is chipped and peeling, so Mr. Harris is hoping for a beautiful new sign to attract business.

Jose loves to paint and design things so he is the right person for the job. Jose is excited. He takes down the old sign and begins thinking about how he is going to design it.

Here is some of the information that Jose has to work with.

- The original sign is 4.25' $\times$ 2.5'
- The letters on the original sign are 1.67' high

While Jose is working on his drawing, Mr. Harris walks up behind him.

"Jose, I think we should work with a new sign board too. Please round the length of the sign to the nearest half foot and the width to the nearest whole foot. Also, please make the letters a bit larger than the original. Maybe round up to the nearest foot on those too," Mr. Harris says to Jose with a twinkle in his eye.

Jose smiles and Mr. Harris and then shrugs when Mr. Harris walks away.

First, underline all of the important information. This has been done above.

There are two parts to Jose's sign dilemma.

The first part is to round the length to the nearest half foot and the width of the original sign to the nearest foot.

Let's look at the dimensions of the original sign: 4.25' $\times$ 2.5'.

We want to round the length to the nearest half foot: 4.25 rounds to 4.5. Because the nearest half foot to .25 is .50.

The new length of the sign is 4.5'.

Next, we look at the width of the sign.

We want to round the width to the nearest foot, so we round 2.5' to 3 feet.

The new width of the sign is 3 feet.

Jose has been having a trickier time with the sizing of the letters. The current size of the letters is 1.67'. He needs to round it to the nearest foot.

Let's look at the decimal part of the measurement.

.67 is closer to one whole than to .50, so we round up.

This is actually quite simple. The question is whether 1.67 is closer to 1 or to 2. If we use the trick we have been practicing and look at the decimal along as if it were a whole number, then the question becomes: Is 67 closer to 0 or to 100? Since 67 is obviously closer to 100, .67 is closer to 1. Since we have already 1 whole, we add 1 more whole, and as a result, 1.67 feet rounds to 2 feet.

You can use the rules for rounding whenever you are rounding any decimal.

Vocabulary

Here are the vocabulary words that you will find throughout this lesson.

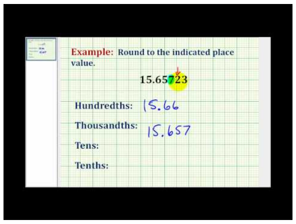
Round
to use place value to change a number whether it is less than or greater than the digit in the number

Decimal
a part of a whole written to the right of a decimal point. The place value of decimals is marked by THS (such as tenTHS, hundredTHS, etc).

Leading Digit
the first digit of a tiny decimal that is not a zero

Small decimals
decimals that have several zeros to the right of the decimal point before reaching a number.

Technology Integration



MEDIA

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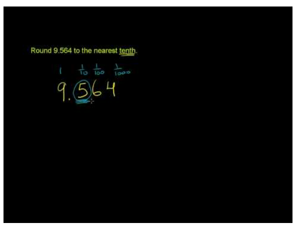
URL: http://www.ck12.org/flx/render/embeddedobject/5321

James Sousa, Rounding Decimals



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Click image to the left or use the URL below.

URL: http://www.ck12.org/flx/render/embeddedobject/5322

Khan Academy Rounding Decimals

Other Videos:

This video shows two students in the sixth grade explaining how to round decimals.

http://www.mathtrain.tv/play.php?vid=84

Time to Practice

Directions: Use the number line and round to the nearest decimal on the number line.

1. 2.54
2. 2.12
3. 2.78
4. 2.89
5. 2.33
6. 2.42
7. 2.97
8. 2.01
9. 2.11
10. 2.27

Directions: Round according to place value

11. Round .45 to the nearest tenth
12. Round .67 to the nearest tenth
13. Round .123 to the nearest tenth
14. Round .235 to the nearest hundredth
15. Round .567 to the nearest hundredth
16. Round .653 to the nearest hundredth
17. Round .2356 to the nearest thousandth
18. Round .5672 to the nearest thousandth
19. Round .8979 to the nearest thousandth
20. Round .1263 to the nearest thousandth

Directions: Round each to the leading digit.

21. .0045
22. .0067
23. .000546
24. .000231
25. .000678
26. .000025
27. .000039
28. .000054
29. .0000278
30. .0000549

Directions: Round each number to the specified place value.

31. 5,689,123 to the nearest million
32. 456,234 to the nearest ten thousand
33. 678,123 to the nearest thousand
34. 432,234 to the nearest hundred thousand
35. 567,900 to the nearest thousand

1.5 Decimal Estimation

Introduction

Recycling

Jose has had many new ideas for improving life at the "Add It Up Ice Cream Stand." His newest idea focuses on recycling.

In addition to ice cream, the stand also sells sodas that are packaged in aluminum cans. Because you can turn in cans for recycling and receive some money back, Jose thinks that this could be a way for the ice cream stand to generate a little more income.

He explained his idea to Mr. Harris who loved the concept. Jose put out recycling bins the first week of June. On the last day of each month, Jose took the recycled cans to the recycling center and collected money on his returns. He decided to keep track of the additional income in a small notebook.

Here is what Jose collected in June, July and August.

June $25.77

July $33.45

August $47.62

Julie asks Jose about how much he has made in recycling.

She also wants to know about how much more he made in August versus June.

Jose looks at his notebook and just by looking at the numbers can't remember how to estimate.

The decimals are throwing him off.

You can help Jose, by the end of the lesson you will know how to estimate sums and differences of decimals in a couple of different ways.

Pay attention, we will return to this dilemma at the end of the lesson.

What You Will Learn

In this lesson, you learn the following skills.

- Estimate sums and differences of decimals using rounding
- Estimate sums and differences of decimal numbers using front –end estimation
- Compare results of different estimation methods
- Approximate solutions to real-world problems using decimal estimation

Teaching Time

I. **Estimate Sums and Differences of Decimals Using Rounding**

Do you remember what it means to *estimate* ?

To estimate means to find an answer that is close to but not exact. It is a reasonable answer to a problem.

What does the word *sum* and the word *difference* mean?

If you think back, you will remember that you have already been introduced to the word sum and the word difference. A sum is the answer from an addition problem. The word difference is the answer of a subtraction problem.

How can we estimate a sum or a difference when our problem has decimals?

The easiest way to estimate a sum or a difference of decimals is to round the decimal.

If we round the decimal to the nearest whole number, we can complete the problem using mental math or at least simplify the problem so that finding an answer is easier.

Let's look at an example.

Example

Estimate 15.7 + 4.9 = _____

In this problem, we only want to estimate our sum. Therefore, we can use our rules for rounding decimals to help us round each decimal to the nearest whole number.

15.7, the place being rounded is the 5, we look at the 7 and round up.

15.7 becomes 16

4.9, the place being rounded is the 4, we look at the 9 and round up.

4.9 becomes 5

Next, we rewrite the problem.

16 + 5 = 21

Our answer is 15.7 + 4.9 = 21.

We can also use rounding when estimating sums of larger numbers.

Example

Estimate 350.12 + 120.78 = _____

We round each to the nearest whole number to find a reasonable estimate.

350.12 becomes 350.

120.78 becomes 121.

350 + 121 = 471

Our answer is 350.12 + 120.78 = 471.

What about differences in estimations with subtraction?

We can work on these problems in the same way, by rounding.

Example

Estimate 45.78 - 22.10 = _____

45.78 rounds to 46.

22.10 rounds to 22.

46 - 22 = 24

Our answer is 45.78 - 22.10 = 24.

Can we use rounding to estimate sums and differences that involve money?

Of course!! Look at this example and see how it is done.

Example

Estimate $588.80 - $310.11 = _____

$588.80 becomes 589 *we can leave off the zeros to make it simpler to estimate*

$310.11 becomes 310

589 - 310 = 279

Our answer is $588.80 - $310.11 = $279.00.

Now it is time for you to try a few on your own. Estimate each sum or difference using rounding.

1. **2.67 + 3.88 + 4.10 =** _____
2. **56.7 - 22.3 =** _____
3. **$486.89 - $25.22 =** _____

Take a minute to check your work with a peer.

II. **Estimate Sums and Differences of Decimals Using Front –End Estimation**

We can also estimate using something called ***front –end estimation***.

Front –end estimation is a useful method of estimating when you are adding or subtracting numbers that are greater than 1000.

Here are the steps for front –end estimation.

1. **Keep the digits of the two highest place values in the number.**
2. **Insert zeros for the other place values.**

Now, let's apply this to a problem.

Example

Estimate 4597 + 3865 = _____

We follow the rules for front –end estimation since each number is over 1000.

4597 becomes 4500. 4 and 5 are the digits of the two highest place values and we filled in zeros for the rest of the places.

3865 becomes 3800. 3 and 8 are the digits of two highest place values and we filled in zeros for the rest of the places.

Now we can rewrite the problem.

4500 + 3800 = 8300

Our answer for 4597 + 3865 is 8300.

What about a problem where we have one number over 1000 and one number not over 1000?

We can use front –end estimation for the number over 1000, and we can round to the highest place value for the number under 1000.

Example

Estimate 4496 - 745 = _____

4496 becomes 4400 using front –end estimation.

745 becomes 700 by rounding to the nearest hundred.

4400 - 700 = 3700

Our answer for 4496 - 745 is 3700.

Use front –end estimation on your own to estimate the following problems.

1. **5674 + 1256 =** _____
2. **4632 - 576 =** _____
3. **8932 + 1445 =** _____

Check your answers with a neighbor. Are your estimations reasonable?

Write down a few notes on front –end estimation before continuing on.

What about front–end estimation and decimals?

When using front –end estimation and decimals, we figure out how to keep the wholes separate from the parts and then combine them together.

Here are the steps to front –end estimation with decimals.

1. **Add the front digits of the numbers being added or subtracted.**
2. **Round off the decimals of the numbers being added or subtracted.**
3. **Combine or subtract the results.**

Wow! That sounds confusing. Let's walk through it by using an example.

Example

2.10 + 3.79 = _____

We start with the front digits of the numbers being added. That means we add 2 + 3 = 5.

Next, we round the decimal part of each number. .10 stays .10 and .79 becomes .80

.80 + .10 = .90

Now we add, since that is the operation, the two estimates together.

5 + .90 = 5.90

Our answer for 2.10 + 3.79 is 5.90.

Here is a subtraction example.

Example

16.79 - 14.12 = _____

We start by subtracting the front ends. 16 - 14 = 2

Next, we round the decimal parts. .79 becomes .80 and .12 becomes .10.

Subtract those decimals .80 - .10 = .70.

Combine for the answer = 2.70.

Our answer for 16.79 - 14.12 is 2.70.

Now it is time for you to try a few on your own. Use front –end estimation here.

1. **54.77 + 22.09 =** _____
2. **18.22 + 19.76 =** _____

Take a minute to check your work with a peer.

III. **Compare the Results of Different Estimation Methods**

Now that you have learned two different ways of estimating sums and difference, how can you decide which method is the better method?

Remember that a method is best if it provides the answer that is the most reasonable.

Let's look at a few examples, use both methods of estimation and decide which method gives us the answer that makes the most sense.

Example

57.46 + 18.21 = _____

Now let's apply what we have learned about estimation to the problem above.

We are going to use front –end estimation first and then we'll apply estimating by rounding.

Here is our work for front –end estimation.

57 + 18 = 75 ***Now we have added the fronts***

.46 becomes .50, .21 becomes .20 and .50 + .20 = .70

Put it altogether, 75 + .70 = 75.70

Now let's see what our answer is if we use rounding.

57.46 rounds to 57

18.21 rounds to 18

Our answer is 57 + 18 = 75

How can we tell which one is the most accurate method of estimation?

Let's see what the actual answer would be. Then we can figure out which method of estimation got us closer to the actual answer.

57.46 + 18.21 = 75.67

Wow! When we used front –end estimation, our answer was 75.70. That is very close to 75.67. Our other answer would have gotten us into the ball park, but wasn't as close to the actual answer.

Sometimes, one method of estimation is better than the other. We have to look at each problem individually to figure this out. For the example that we just finished, the best choice of estimation would be front –end estimation.

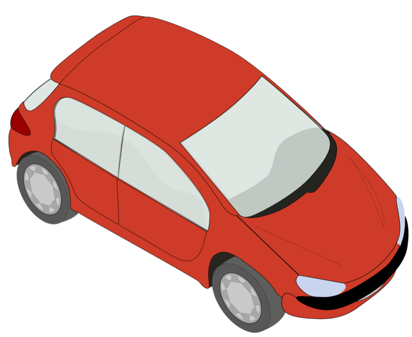
What type of problem would be better for rounding?

Rounding is best when working with very large numbers. Then we can get an estimate of the answer without dealing with all of the fronts and ends of numbers using front –end estimation.

Let's look at an example to help us understand this.

Example

$6927.11



$8100.89

Here are two cars that are for sale.

The first car has a price tag of $6927.11.

The second car has a price tag of $8100.89.

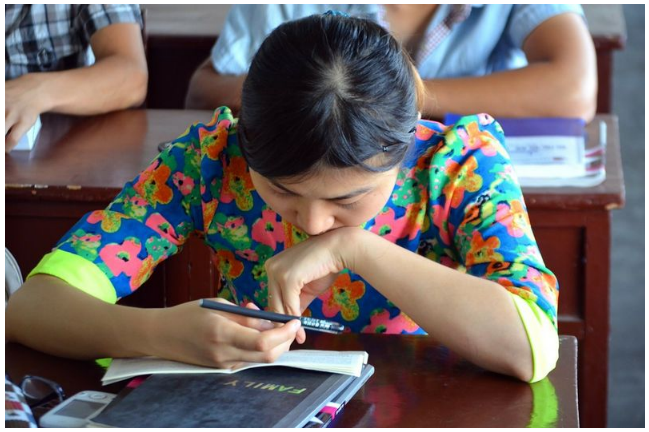
Let's say that we wanted to figure out the difference between the prices of these two cars. If we just wanted to get an idea of how much one car was versus the other, we can estimate and the difference.

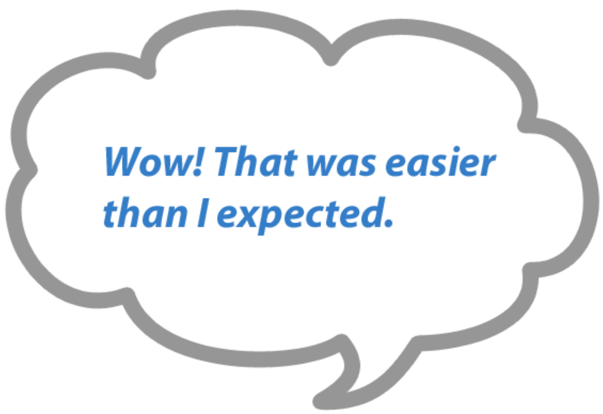
Let's use rounding to figure out the difference between car 1 and car 2.

Car 1 $6927.11 rounds to $7000.00

Car 2 $8100.89 rounds to $8100.00

There is a difference of about $1100.00 between the two cars.

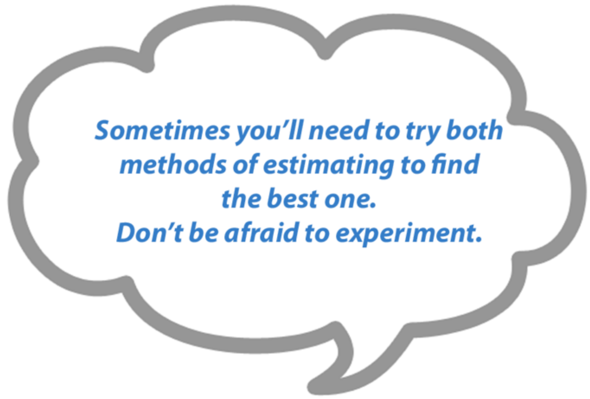




Let's see if it was as easy with front –end estimation.

First, add the front ends. 6927 + 8100........

For this problem, because of its large numbers, it makes much more sense to round each number. Using front –end estimation would have required us to add each number and then round and add the decimal parts. It definitely would have been more challenging.



Real Life Example Completed

Recycling

You have learned all about front –end estimation and rounding to estimate sums and differences.

Now we are ready to help Jose sort through his recycling dilemma.

Let's take another look at the problem.

Jose has had many new ideas for improving life at the "Add It Up Ice Cream Stand." His newest idea focuses on recycling.

In addition to ice cream, the stand also sells sodas that are packaged in aluminum cans. Because you can turn in cans for recycling and receive some money back, Jose thinks that this could be a way for the ice cream stand to generate a little more income.

He explained his idea to Mr. Harris who loved the concept. Jose put out recycling bins the first week of June. On the last day of each month, Jose took the recycled cans to the recycling center and collected money on his returns. He decided to keep track of the additional income in a small notebook.

Here is what Jose collected in June, July and August.

June $25.77

July $33.45

August $47.62

Julie asks Jose about how much he has made in recycling.

She also wants to know about how much more he made in August versus June.

Jose looks at his notebook and just by looking at the numbers can't remember how to estimate.

The decimals are throwing him off.

First, let's go through and underline all of the important information.

The next thing that we need to do is to estimate the sum of the amounts of money that Jose collected in June, July and August.

Let's start by rounding.

$25.77 becomes $26.00

$33.45 becomes $33.00

$47.62 becomes $48.00

Our estimated sum is $107.00.

After rounding, Jose decides to try front –end estimation to see if he can get an even more accurate estimate of the sum.

First, add the front ends, 25 + 33 + 47 = 105.

Next round the decimal parts and add them, .77 = .80, .45 = .50, .62 = .60.

$$.80+.50+.60 = 1.90$$
$$105+1.90 = \$106.90$$

Jose shows his work to Julie and the two of them are amazed! The answers for both methods of estimation were definitely very close!

Next, Jose works to figure out the difference between the amount of money collected in June versus August.

Since both sums were similar, he decides to use rounding to estimate this difference.

June = $25.77 which rounds to $26

August = $47.62 which rounds to $48

48 - 26 = $22.00

"Congratulations Jose! Your recycling campaign is definitely working! Keep up the good work," Julie says to Jose after seeing his results.

Jose feels proud because of his accomplishment. The recycling campaign will remain at the ice cream stand.

Vocabulary

Here are the vocabulary words from this lesson.

Estimate

to find an answer that is reasonable and close to an exact answer.

Sum
the result of an addition problem

Difference
the result of a subtraction problem

Front end estimation
estimating by adding the front ends of each number in the problem, then rounding and adding the decimal parts of each number.

- Works well with smaller numbers

Rounding
converting a number to its nearest whole number.

- Works well with larger numbers

Time to Practice

Directions: Estimate each sum or difference by rounding.

1. 56.32 + 23.12 = _____
2. 18.76 + 11.23 = _____
3. 14.56 + 76.98 = _____
4. 11.12 + 54.62 = _____
5. 33.24 + 45.32 = _____
6. 18.97 + 15.01 = _____
7. 22.43 + 11.09 = _____
8. 4.52 + 3.21 = _____
9. 19.19 + 27.75 = _____
10. 87.12 + 88.90 = _____
11. 67.19 - 33.12 = _____
12. 88.92 - 33.10 = _____
13. 76.56 - 3.45 = _____
14. 65.72 - 11.12 = _____
15. 77.34 - 43.02 = _____
16. 88.02 - 11.10 = _____
17. 89.32 - 18.03 = _____
18. 24.67 - 10.10 = _____

19. 37.82 - 14.20 = _____

20. 55.88 - 44.22 = _____

21. 334.56 - 125.86 = _____

22. 456.11 + 112.18 = _____

Directions: Estimate using front –end estimation.

23. 34.66 + 11.12 = _____

24. 43.18 + 16.75 = _____

25. 2.34 + 1.56 = _____

26. 7.89 + 5.79 = _____

27. 8.90 + 3.21 = _____

28. 7.18 - 3.13 = _____

29. 12.65 - 7.23 = _____

30. 15.70 - 11.10 = _____

31. 25.67 - 18.40 = _____

32. 78.46 - 55.21 = _____

33. 88.12 - 34.06 = _____

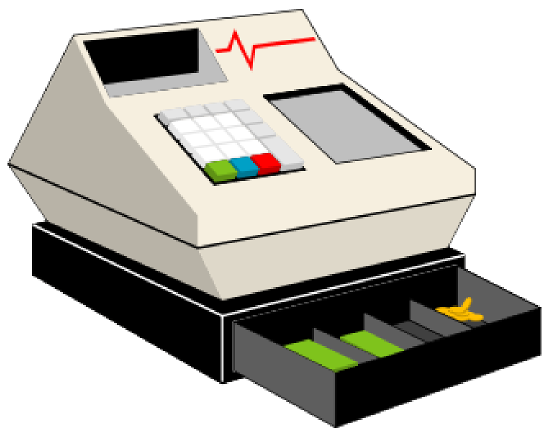
34. 87.43 - 80.11 = _____

35. 94.12 - 7.08 = _____

1.6 Adding and Subtracting Decimals

Introduction

The Broken Cash Register



When Julie arrived for her shift at the ice cream stand, she was surprised to find out that the cash register was broken.

"You can just figure out each total and the customer's change," Mr. Harris said to Julie with a smile.

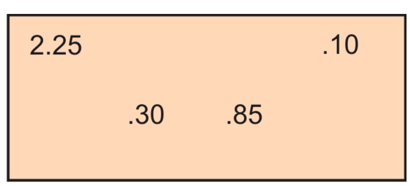
Julie grimaced as she got out a pad of paper and pencil. She knew that she was going to need to do some quick addition and subtraction to make this whole day work.

Very soon her first customer arrived. This customer ordered a small cone for $2.25 and gave Julie exact change.

"Maybe this won't be so tough after all," Julie thought.

Then her luck ended. A woman arrived and ordered a small cone with sprinkles, caramel, and an extra scoop of ice cream.

Julie quickly jotted the following numbers down on a piece of paper.



While Julie was working to figure out the sum, the woman handed Julie a $10.00 bill and two quarters.

"I am so glad that I have the change," she said to Julie.

Julie frantically began to work out the math on her piece of paper.

How can Julie add up the decimals?

Is there a way for her to do it mentally?

What about the customer's change? If the woman gave Julie a ten dollar bill and two quarters, how much change should she get back?

This lesson is going to teach you all about adding and subtracting decimals.

Hold on Julie, help is right around the corner!!

What You Will Learn

In this lesson, you will learn the following skills:

- Adding and Subtracting Decimals by rewriting with additional zero place holders.
- Using mental math to add/subtract decimals
- Identifying the commutative and associative properties of addition in decimal operations, using numerical and variable expressions
- Solving real world problems involving decimal addition and subtraction

Teaching Time

In our last lesson we learned how to estimate the sums and differences of problems with decimals. Remember, an estimate only works when we don't need an exact answer.

Let's think about Julie. She can't use an estimation to solve her problem. She needs to know the exact cost of the ice cream cone with all of the additions and she needs to know the exact change to give back to the customer. Think about how funny it would be if Julie told her what an estimate of the cost would be and if she gave back an estimate of the change.

In problems like Julie's situation, we need to know how to add and subtract decimals.

Let's begin by learning how to find an exact sum or an exact difference.

I. **Adding and Subtracting Decimals by Rewriting With Additional Zero Place Holders**

To add or subtract decimals, we are going to be working with the wholes and parts of the numbers separately.

We want to add or subtract the parts and then add or subtract the wholes.

How can we do this?

The best way to do this is to keep the parts together and keep the wholes together.

To do this, we simply line up the decimal points in each number that we are adding or subtracting.

Let's look at an example.

Example

Add 3.45 + 2.37 = _____

In this problem we have parts and wholes. Let's rewrite the problem vertically, lining up the decimal points.

$$\begin{array}{r} 3.45 \\ \underline{+\ 2.37} \end{array}$$

Next, we can add the columns vertically and bring the decimal point down into the answer of the problem.

$$\begin{array}{r} 3.45 \\ \underline{+\ 2.37} \\ 5.82 \end{array}$$

Our answer is 5.82.

Does this work the same way when finding a difference?

Yes. We can line up the decimals in a subtraction problem and subtract the digits the same way.

Example

6.78 - 2.31 = _____

First, we line up the problem vertically.

$$\begin{array}{r} 6.78 \\ -\ 2.31 \\ \hline \end{array}$$

Next, we subtract each digit vertically.

$$\begin{array}{r} 6.78 \\ -\ 2.31 \\ \hline 4.47 \end{array}$$

Our answer is 4.47.

These examples both had the same number of digits in them. They each had one whole number and a decimal in the hundredths.

What happens when you have decimals with different numbers of digits in them?

When we have a problem like this, we still line up the decimal points, but we add zeros to help hold places where there aren't numbers. This helps us to keep our addition and subtraction straight.

Let's look at an example.

Example

5 + 3.45 + .56 = _____

First, we line up the problem vertically.

Remember that the decimal point in a whole number is after the number

$$\begin{array}{r} 5.00 \\ 3.45 \\ +\ 0.56 \\ \hline \end{array}$$

Notice that we added in zeros to help hold places where we did not have numbers. Now each number in the problem has the same number of digits. We can add them with ease.

$$\begin{array}{r} 5.00 \\ 3.45 \\ +\,0.56 \\ \hline 9.01 \end{array}$$

Our answer is 9.01.

We can do the same thing with a subtraction problem. We add zeros to help hold places where there are not digits. That way each number has the same number of places.

Example

67.89 - 18.4 = _____

First, we line up the problem vertically with the decimal point.

$$\begin{array}{r} 67.89 \\ -\,18.40 \\ \hline 49.49 \end{array}$$

Our answer is 49.49.

Now it is time for you to try a few on your own.

1. **4.56 + .89 + 2.31 = _____**
2. **16 - 12.22 = _____**
3. **88.92 + .57 + 3.12 = _____**

Take a few minutes to check your addition and subtraction with a peer. Did you remember to add in the zeros for place holders?

II. **Use Mental Math to Add/Subtract Decimals**

Sometimes, you don't need to go through all of the work of lining up decimal points and filling in the zeros. Sometimes you can use mental math to figure out a sum.

When is mental math most helpful with decimal sums and differences?

When you have a decimal where the decimal parts can easily add up to be one whole, you can use mental math to figure out the sum.

Think about this. If you had .30 + .70, you know that 3 + 7 is 10, therefore you know that .30 + .70 is 1.00.

Let's apply this information.

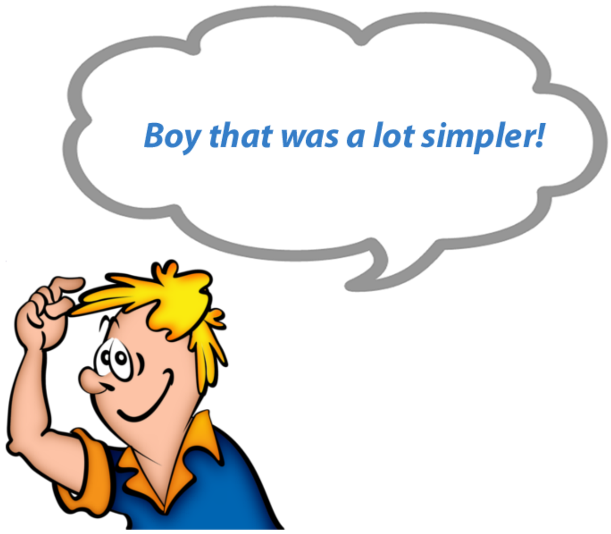
Example

5.30 + 6.70 = _____

Here we can start by looking at the decimals, since .30 + .70 is 1. Then we combine the whole numbers and add the total of the decimals to get an answer:

5 + 6 = 11 + 1 = 12

Our answer is 12.



What about subtraction?

We can use mental math to complete subtraction problems too.

We just look for which decimals add up to be wholes and go from there.

Let's look at an example.

Example

25.00 - 22.50 = _____

We are subtracting 25.00 - 22.50, we can think about this problem in reverse to make the mental math simpler.

"What plus 22.50 will give us 25.00?" Think: 2.50 plus what equals 5.00?

25.00 - 22.50 = 2.50

Our answer is 2.50.

Not all problems will be able to be solved mentally, but when we can mental math makes things a whole lot simpler!!

Here are few for you to work on. Add or subtract using mental math.

1. **33.50 + 5.50 = _____**
2. **10 - 3.75 = _____**
3. **18.25 + 2.25 = _____**

Take a few minutes to check your work with a peer. Do your answers match? If so, move on. If not, recheck your work.

III. **Identify and Apply the Commutative and Associative Properties of Addition in Decimal Operations**

We have just learned how to add and subtract decimals both by using mental math and by completing the arithmetic on a piece of paper by lining up the decimal points.

We can also apply two ***properties*** to our work with decimals.

A property is a rule that remains true when applied to certain situations in mathematics.

We are going to work with two properties in this section, ***the associative property and the commutative property***.

Let's begin by learning about the *commutative property* .

The commutative property means that you can switch the order of any of the numbers in an addition or multiplication problem around and you will still receive the same answer.

Here is an example.

Example

4 + 5 + 9 = 18 is the same as 5 + 4 + 9 = 18

The order of the numbers being added does not change the sum of these numbers. This is an example of the commutative property.

How can we apply the commutative property to our work with decimals?

We apply it in the same way. If we switch around the order of the decimals in an addition problem, the sum does not change.

Example

4.5 + 3.2 = 7.7 is the same as 3.2 + 4.5 = 7.7

Now we can look at the *associative property* .

The associative property means that we can change the groupings of numbers being added (or multiplied) and it does not change the sum. This applies to problems with and without decimals.

Example

(1.3 + 2.8) + 2.7 = 6.8 is the same as 1.3 + (2.8 + 2.7) = 6.8

Notice that we use parentheses to help us with the groupings. When we regroup numbers in a different way the sum does not change.

What about variables and decimals?

Sometimes, we will have a problem with a variable and a decimal in it. We can apply the commutative property and associative property here too.

Example

$x+4.5$ is the same as $4.5+x$

$(x+3.4)+5.6$ is the same as $x+(3.4+5.6)$

The most important thing is that the order of the numbers and the groupings can change but the sum will remain the same.

Look at the following examples and name the property illustrated in the example.

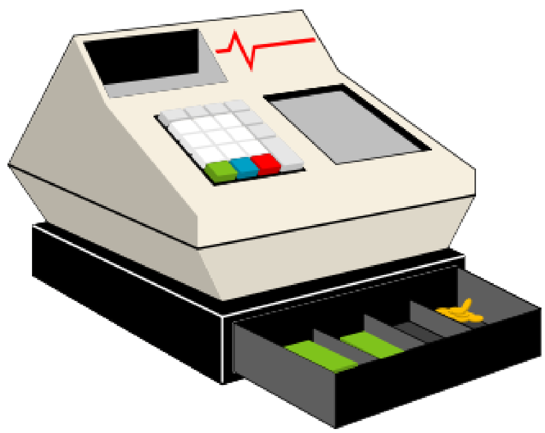
1. **3.4 + 7.8 + 1.2 = 7.8 + 1.2 + 3.4**
2. **(1.2 + 5.4) + 3.2 = 1.2 + (5.4 + 3.2)**

3. $x+5.6+3.1=3.1+x+5.6$

Check your work with a peer. Did you name the correct property?

Real Life Example Completed

The Broken Cash Register



Alright Julie, help is now on the way.

Now that we have learned how to add and subtract decimals, we are ready to help Julie with her customer.

Let's look at the problem once again.

When Julie arrived for her shift at the ice cream stand, she was surprised to find out that the cash register was broken.

"You can just figure out each total and the customer's change," Mr. Harris said to Julie with a smile.

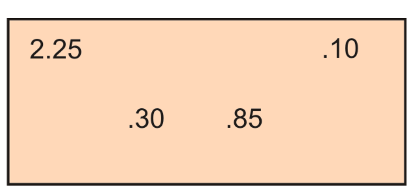
Julie grimaced as she got out a pad of paper and pencil. She knew that she was going to need to do some quick addition and subtraction to make this whole day work.

Very soon her first customer arrived. This customer ordered a small cone for $2.25 and gave Julie exact change.

"Maybe this won't be so tough after all," Julie thought.

Then her luck ended. A woman arrived and ordered a small cone with sprinkles, caramel, and an extra scoop of ice cream.

Julie quickly jotted the following numbers down on a piece of paper.



While Julie was working to figure out the sum, the woman handed Julie a $10.00 bill and two quarters.

"I am so glad that I have the change," she said to Julie.

Julie frantically began to work out the math on her piece of paper.

How can Julie add up the decimals?

Is there a way for her to do it mentally?

What about the customer's change? If the woman gave Julie a ten dollar bill and two quarters, how much change should she get back?

First, let's underline all of the important information.

Next, we need to figure out the cost of the ice cream cone.

Here are the numbers that Julie wrote down.

2.25 + .10 + .30 + .85 = _____

Next, we need to line up the numbers vertically.

$$\begin{array}{r} 2.25 \\ .10 \\ .30 \\ +\ .85 \\ \hline 3.50 \end{array}$$

The cost of the ice cream cone is $3.50.

Julie took the ten dollar bill and the two quarters from the customer.

We can use mental math to figure out the customer's change.

$$\$10.50 - 3.50 = \_\_\_\_$$
$$.50 - .50 = 0$$
$$10 - 3 = 7$$

Julie confidently handed the customer $7.00 in change. The customer smiled, thanked Julie and left eating her delicious ice cream cone.

Vocabulary

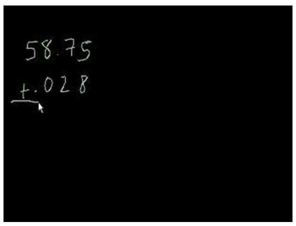
Here are the vocabulary words in this lesson.

Properties
: the features of specific mathematical situations.

Associative Property
a property that states that changing the grouping in an addition problem does not change the sum.

Commutative Property
a property that states that changing the order of the numbers in an addition problem does not change the sum.

Technology Integration



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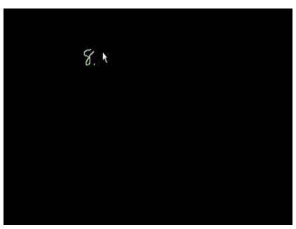
Khan Academy Adding Decimals



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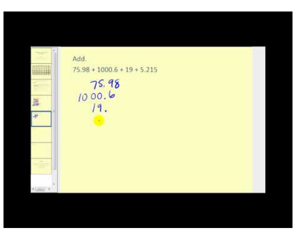
Khan Academy Subtracting Decimals



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James Sousa, Adding and Subtracting Decimals

Other Videos:

1. http://www.gamequarium.org/cgi-bin/search/linfo.cgi?id=7544 –Blackboard video on how to add decimals.
2. http://www.gamequarium.org/cgi-bin/search/linfo.cgi?id=7545 –Blackboard video on how to subtract decimals.

Time to Practice

Directions: Add or subtract the following decimals.

1. 4.5 + 6.7 = _____
2. 3.45 + 2.1 = _____
3. 6.78 + 2.11 = _____
4. 5.56 + 3.02 = _____
5. 7.08 + 11.9 = _____
6. 1.24 + 6.5 = _____
7. 3.45 + .56 = _____
8. 87.6 + 98.76 = _____
9. 76.43 + 12.34 = _____
10. 5 + 17.21 = _____
11. 17.65 - 4 = _____
12. 18.97 - 3.4 = _____
13. 22.50 - .78 = _____
14. 27.99 - 1.99 = _____
15. 33.11 - 3.4 = _____
16. 44.59 - 11.34 = _____
17. 78.89 - 5 = _____
18. 222.56 - 11.2 = _____
19. 567.09 - 23.4 = _____
20. 657.80 - 3.04 = _____

Directions: Use mental math to compute each sum or difference.

21. .50 + 6.25 = _____
22. 1.75 + 2.25 = _____
23. 3.50 + 4.50 = _____
24. 7.25 + 1.25 = _____
25. 8.75 + 3.25 = _____
26. 8.50 - 2.50 = _____
27. 10 - 4.50 = _____
28. 12 - 3.75 = _____
29. 15.50 - 5.25 = _____
30. 20 - 15.50 = _____

1.7 Stem-and-Leaf Plots

Introduction

Ice Cream Counts

The "Add It Up Ice Cream Stand" has had an excellent summer. Mr. Harris told all of his employees that he is thrilled with the number of ice cream cones that were sold each day.

The last week of August was the most successful week of sales. Here are the counts that Mr. Harris collected on each day during this last week of August.

Mon - 78

Tues - 86

Wed - 52

Thurs - 67

Fri - 70

Sat - 75

Sun - 78

Julie wants to design a beautiful chart to give to Mr. Harris as a gift to show the best sales for the week.

"Why don't you put those in a stem-and-leaf plot," Jose suggests when Julie tells him the idea.

"Good idea," Julie says and she gets to work.

Now it is your turn. You are going to make a stem-and-leaf plot to show Mr. Harris' ice cream sales for his best week ever.

The title of the stem-and-leaf plot is "THE BEST WEEK EVER."

Pay attention throughout this lesson so that you can build a stem-and-leaf plot to organize the data.

What You Will Learn

In this lesson you will learn the following skills.

- Organize a set of data in a stem-and-leaf plot.
- Use a stem-and-leaf plot to find the range of a set of data.
- Use a stem-and-leaf plot to find the mean, median and mode of a set of data.

Teaching Time

I. **Organize a Set of Data in a Stem-and-Leaf Plot**

A ***stem-and-leaf*** plot is a visual diagram where you organize numbers according to place value. The ***data*** is organized in either ***ascending or descending*** order. To build a stem-and-leaf plot, we use place value as our method of organizing data.

If we had a 15 as our number, the stem would be a ten since that is the tens place value. The leaf would be the 5.

To write it as a stem-and-leaf plot, here is what it would look like.

1 | 5 **This means 15.**

A stem-and-leaf plot is most useful when looking at a series of data. When we have a series of data, we can organize them according to place value.

Let's look at an example.

Example

22, 15, 11, 22, 24, 33, 45

Let's say that we want to organize this data in a stem-and-leaf plot.

First, we organize them by the tens place since all of our numbers have tens places as the highest place value.

11, 15, 22, 22, 24, 33, 45

Next, we put each stem on the left side of our vertical line.

1	
2	
3	
4	

Notice that the largest of each place is on the left of the lines. Now we can put the ones or the stems on the right of the vertical line.

1	1 5
2	2 2 4
3	3
4	5

Each number in the data has been organized. The tens place is on the left for each number and the ones places that go with each ten are on the right side of the vertical bar.

This is our completed stem-and-leaf plot.

Helpful Hint 1

Notice that we list repeated values in the chart.

Let's look at another example.

Example

33, 34, 36, 45, 40, 62, 67, 68

We start by organizing the stems separate from the leaves.

3	3 4 6
4	0 5
5	
6	2 7 8

Notice that there isn't a number in the fifties in the list of data.

We still need to include it in the stem-and-leaf plot. Because of this, we can leave the leaf empty, but we still include the stem.

Helpful Hint 2

List stems that are between numbers even if they don't have leaves

Include zeros in the leaves for numbers that end in 0

Now that we know how to create a stem-and-leaf plot, how can we interpret the data?

Each stem and set of leaves creates an ***interval***.

Let's look at the intervals for the stem-and-leaf plot we just created.

3	3 4 6
4	0 5
5	
6	2 7 8

The interval for the 30's is 33 - 36.

The interval for 40's is 40 - 45.

The interval for 60's is 62 - 68.

Practice what you have learned. Go ahead and create a stem-and-leaf plot from the following data set.

1. **11, 10, 13, 22, 25, 30, 32, 44, 46, 47, 52, 53, 55, 72**

Take a minute to check your work with a neighbor. Did you include a stem of 6?

II. **Use a Stem-and-Leaf Plot to Find the Range of a Set of Data**

Think back to our work on data. What is the ***range***?

The range is the difference between the maximum score and the minimum score.

We can use a stem-and-leaf plot to find the range of a set of data.

Let's look at the following example.

Example

2	2 3 4 4 5
3	3 4 5
4	
5	1 1 2 3 4 5

The smallest number in the stem-and-leaf plot is 22. You can see that by looking at the first stem and the first leaf.

The greatest number is the last stem and the last leaf on the chart. In this case, the largest number is 55.

To find the range, we subtract the smallest number from the largest number. This difference will give us the range.

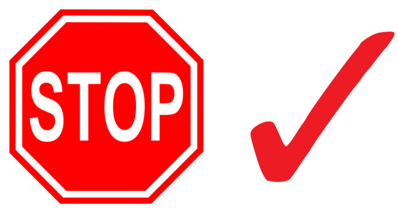
55 - 22 = 33

The range is 33 for this set of data.

Look at the following stem-and-leaf plot and answer these questions.

1	2 3 4
2	3 4 5 6 7
3	0 1 2
4	2 4 5 6 7

1. **What is the range for this data set?**
2. **What is the smallest interval?**
3. **What is the largest interval?**



How did you do? Is the range accurate? Check your work with a friend.

III. **Use a Stem-and-Leaf Plot to Find Mean, Median and Mode of a Set of Data**

Remember back to our chapter on data?

We worked with data sets and found the mean, median and mode of each set of data.

The *mean* is the average of a set of data.

The *median* is the middle number of a set of data.

The *mode* is the number that occurs the most in a set of data.

We can use a stem-and-leaf plot to find the mean, median and mode of a set of data.

Let's look at an example.

Example

3	5 6 7 8
4	0 0 1 2 3
5	5 5 5 6 7 8 9

Here we have a data set with numbers that range from 35 to 59.

The largest interval is from 55 to 59.

The smallest interval is from 35 to 38.

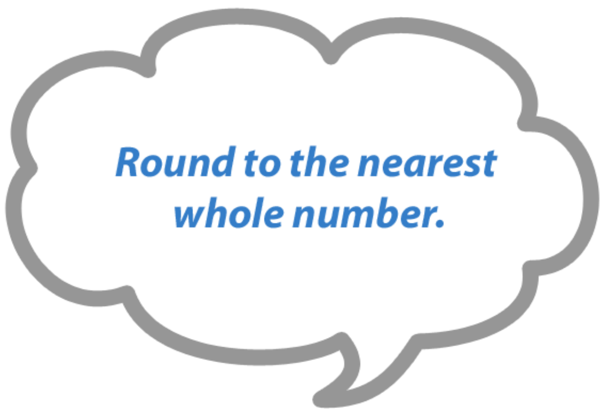
What is the mean for this set of data?

To find the mean, we add up all of the numbers in the set and divide by the number of values that we added.

35 + 36 + 37 + 38 + 40 + 40 + 41 + 42 + 43 + 55 + 55 + 55 + 56 + 57 + 58 + 59 = 747

We divide by the number of values, which is 16.

$$\frac{747}{16} = 46.68$$



After rounding, our answer is 47.

What is the median for this set of data?

Well, remember that the median is the middle score. We just wrote all of the scores in order from the smallest to the greatest. We can find the middle score by counting to the middle two scores.

42 + 43 These are the two middle scores.

We can find the mean of these two scores and that will give us the median.

42 + 43 = 42.5

The median score is 42.5 for this data set.

What is the mode for this data set?

The mode is the value that appears the most.

In this set of data, 55 is the number that appears the most.

The mode is 55 for this data set.

Real Life Example Completed

Ice Cream Counts

The "Add It Up Ice Cream Stand" has had an excellent summer. Mr. Harris told all of his employees that he is thrilled with the number of ice cream cones that were sold each day.

The last week of August was the most successful week of sales. Here are the counts that Mr. Harris collected on each day during this last week of August.

Mon - 78

Tues - 86

Wed - 52

Thurs - 67

Fri - 70

Sat - 75

Sun - 78

Julie wants to design a beautiful chart to give to Mr. Harris as a gift to show the best sales for the week.

"Why don't you put those in a stem-and-leaf plot," Jose suggests when Julie tells him the idea.

"Good idea," Julie says and she gets to work.

The first thing that we are going to do is to organize the data in a stem-and-leaf plot. The smallest stem is 5 and the largest stem is 8.

We can build the stem-and-leaf plot and fill in the stems and the leaves.

5	2
6	7
7	0 5 8 8
8	6

Now we have a stem and leaf plot with the data all arranged.

Use your notebook to answer the following questions on the data.

1. **What is the smallest number of ice cream cones sold?**

2. **What is the largest number of ice cream cones sold?**
3. **What is the range in the number of cones sold?**
4. **What is the interval with the most values in it?**
5. **What is the mode of this data set?**
6. **What is the average number of cones sold?**

Vocabulary

Here are the vocabulary words found in this lesson.

Stem-and-leaf plot
a way of organizing numbers in a data set from least to greatest using place value to organize.

Data
information that has been collected to represent real life information

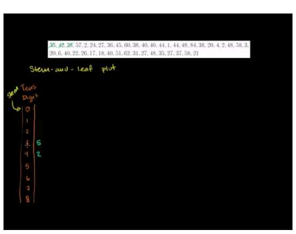
Ascending
from smallest to largest

Descending
from largest to smallest

Interval
a specific period or arrangement of data

Range
the difference from the largest value to the smallest value

Technology Integration



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Khan Academy Stem and Leaf Plots

Other Videos:

1. http://www.mathplayground.com/howto_stemleaf.html –Great video on organizing, building and interpreting a stem and leaf plot.

Time to Practice

Directions: Build a stem-and-leaf plot for each of the following data sets.

1. 42, 44, 45, 46, 51, 52, 53, 60
2. 13, 11, 20, 21, 22, 30, 31, 32
3. 44, 45, 46, 48, 51, 53, 55, 67, 69
4. 10, 19, 19, 10, 11, 13, 14, 14, 15
5. 12, 13, 13, 21, 22, 23, 33, 34, 37, 40
6. 45, 46, 46, 46, 52, 52, 54, 77, 78, 79
7. 60, 60, 62, 63, 70, 71, 71, 88, 87, 89
8. 80, 81, 82, 90, 91, 92, 93, 93, 93, 94
9. 11, 12, 12, 13, 14, 14, 20, 29, 30, 32, 32, 52
10. 33, 45, 46, 47, 60, 60, 72, 73, 74, 88, 89

Directions: Use the stem-and-leaf plots that you created to answer the following questions.

11. What is the range of data in the stem-and-leaf plot in problem 2?
12. What is the mean of the set of data in problem 2?
13. If you round the mean to the nearest whole number, what is the mean now?
14. What is the mode of this data set in problem 2?
15. What is the median number in the data set in problem 2?
16. What is the range of the data in the stem-and-leaf plot in problem 6?
17. What is the mean of this set of data?
18. If we were to round this mean what would the new mean be?
19. What is the mode of this data set?
20. What is the median?

1.8 Use Estimation

Introduction

Summer Job Benefits

Jose has enjoyed working all summer. He loved helping Mr. Harris and his recycling idea ended up being very profitable.

Jose began the summer with an estimate of how much money he thought he would make.

Jose earned $7.00 per hour and he worked ten 30 hour weeks.

Jose ended up earning $2100.00 for the summer, and he is very pleased with his accomplishment.

Now that the summer is over, Jose wishes to spend part of his money on new clothes for school.

He has selected the following items.

$19.95

$32.95

$46.75

Jose brought $100.00 with him to purchase the items.

If he estimates the total cost, what would it be?

How much change will Jose receive from the $100.00?

Using estimation will help Jose with his purchases.

Let's look at some situations where estimation makes the most sense, then we will come back to this problem to help Jose with his shopping.

What You Will Learn

In this lesson, you will learn to use the following skills:

- Read and understand given problem situations
- Develop and use the strategy: Use Estimation
- Plan and compare alternative approaches to solving problems
- Solve real-world problems using selected strategies as part of a plan

Teaching Time

I. **Read and Understand Given Problem Situations**

We can use estimation in several different problem situations. To use estimation, we need to read and understand the problem. There will be clues in the problem to let us know if estimation is a good option for solving that specific problem.

Let's review what it means to estimate.

Estimating means that we are going to be finding an answer that is an approximate answer.

When estimating, our answer must make sense, but it does not need to be exact.

We can find an answer that is reasonable to provide us information for our problem.

When looking at a problem, we need to read the problem to see if estimating is a good option in the problem.

We can look for key words to help us with this.

Here are some of the key words that we use when estimating:

- **Close to**
- **Approximate**
- **Estimate**
- **An answer that makes sense**
- **About**

If you see these words in a word problem, you can use estimating to find the answer.

Let's look at an example.

Example

Kelly wanted to get an idea how much she was spending at the store. On the way to the checkout she looked at the items in her cart. Here are the prices of the food in her cart: $.50, $2.50, $ 3.45 and $ 6.79. About how much will Kelly spend when she checks out?

Are there any key words in this problem?

Yes, the word ABOUT lets us know that we can estimate to find our answer.

Now that we know that we can estimate, how do we use estimation to solve this problem?

II. **Develop and Use the Strategy: Use Estimation**

Once you have figured out that you can estimate to solve the problem, you will need to apply the estimation strategy.

We can do this in one of two ways.

1. Rounding
2. Front –end Estimation.

For the problem that we just looked at, let's use **rounding**.

Here is the problem once again.

Example

Kelly wanted to get an idea how much she was spending at the store. On the way to the checkout she looked at the items in her cart. Here are the prices of the food in her cart, $.50, $2.50, $ 3.45 and $ 6.79. About how much will Kelly spend when she checks out?

Next, let's round each price.

.50 becomes 1

2.50 becomes 3.00

3.45 becomes 3.00

6.79 becomes 7

Now we can add up the rounded answers: 1 + 3 + 3 + 7 = 14

Our answer is $14.00. Kelly will spend approximately $14.00 at the store.

III. **Plan and Compare Alternative Approaches to Solving Problems**

There are many different ways to approach solving a problem. In the last example, we used rounding and estimation. We know that this is an approach that works when we are looking for an approximate answer.

If we had been working with large numbers in the thousands, we would have been using estimation and front –end estimation.

Sometimes, we will need to draw a picture to solve a problem. That is what will make the most sense.

Let's look at an example where we would draw a picture to solve an estimation problem.

Example

Carl is working on building a small cd rack out of wood. He can buy material in a 6' $\times$ 8' rectangular piece of plywood. Carl needs to build two sides from one piece of wood. The sides have the dimensions 2' $\times$ 4'. If Carl buys one sheet of plywood, will he have enough wood for the two sides of the cd rack?

Hmmmm.... How can we work on this problem?

We don't need an exact measurement we just need to know the rough dimensions to figure out if the two sides of the cd rack will fit on piece of plywood.

We can use estimation to do this.

First, let's underline the important information in the problem.

Carl is working on building a small cd rack out of wood. He can buy material in a $\underline{6' \times 8'}$ rectangular piece of plywood. Carl needs to build two sides from one piece of wood. The sides have the dimensions $\underline{2' \times 4'}$. If Carl buys one sheet of plywood, will he have enough wood for the two sides of the cd rack?

Notice here that we show three pictures.

The first one is of the rectangular piece of wood that is 6 $\times$ 8.

The second two are the two rectangles that will make up the side of the cd rack.

This is a visual way to estimate whether the two pieces will fit on the one piece of plywood.

Visually it looks like it will work. Visual estimation is one strategy.

What about if we want to be sure our estimate is accurate?

We can estimate the dimensions of the two sides of the cd rack combined.

$\mathbf{2 \times 4 + 2 \times 4 = 4 \times 4}$

We need a piece of wood that is 4 $\times$ 4 to build the sides of the cd rack.

Since our piece is 6 $\times$ 8 it will work for us.

Our visual estimation is accurate.

Real Life Example Completed

Summer Job Benefits

Now we can help Jose with his shopping. Shopping is a great real life example where estimation is very useful. We can get an idea of how much we are spending as well as about how much change we can receive when estimation.

Let's take another look at the problem.

Jose has enjoyed working all summer. He loved helping Mr. Harris and his recycling idea ended up being very profitable.

Jose began the summer with an estimate of how much money he thought he would make.

Jose earned \$7.00 per hour and he worked ten 30 hour weeks.

Jose ended up earning \$2100.00 for the summer, and he is very pleased with his accomplishment.

Now that the summer is over, Jose wishes to spend part of his money on new clothes for school.

He has selected the following items.

\$19.95

$32.95

$46.75

Jose brought $100.00 with him to purchase the items.

If he estimates the total cost, what would it be?

How much change will Jose receive from the $100.00?

We could use a couple of different strategies to estimate the total of Jose's purchases.

We could use rounding or front –end estimation.

Let's use rounding first.

$19.95 rounds to $20.00

$32.95 rounds to $33.00

$46.75 rounds to $47.00

Our estimate is $100.00.

Hmmm. Ordinarily, rounding would give us an excellent estimate, but in this case our estimate is the amount of money Jose wishes to pay with.

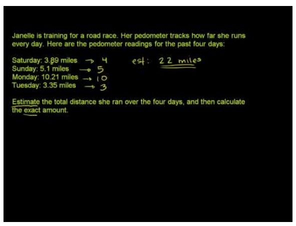
Because of this, let's try another strategy. Let's use front –end estimation and see if we can get a more accurate estimate.

$$19+32+46=97$$
$$1+1+80=2.80$$

Our estimate is $99.80.

With front –end estimation, we can estimate the Jose will receive .20 change from his $100.00. While he isn't going to get a lot of change back, he is going to receive some change so he does have enough money to make his purchases.

Technology Integration



MEDIA

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URL: http://www.ck12.org/flx/render/embeddedobject/54781

Khan Academy Estimation with Decimals

Time to Practice

Directions: Look at each problem and use what you have learned about estimation to solve each problem.

1. Susan is shopping. She has purchased two hats at $5.95 each and two sets of gloves at $2.25 each. If she rounds each purchase price, how much can she estimate spending?

2. If she uses front –end estimation, how does this change her answer?

3. Which method of estimation gives us a more precise estimate of Susan's spending?

4. If she brings $20.00 with her to the store, about how much change can she expect to receive?

5. If she decided to purchase one more pair of gloves, would she have enough money to make this purchase?

6. Would she receive any change back? If yes, about how much?

7. Mario is working at a fruit stand for the summer. If a customer buys 3 oranges at $.99 a piece and two apples for $.75 a piece, about how much money will the customer spend at the fruit stand? Use rounding to find your answer.

8. What is the estimate if you use front –end estimation?

9. Why do you think you get the same answer with both methods?

10. If the customer gives Mario a $10.00 bill, about how much change should the customer receive back?

11. Christina is keeping track of the number of students that have graduated from her middle school over the past five years. Here are her results.

2004 –334

2005 –367

2006 –429

2007 –430

2008 –450

Estimate the number of students who graduated in the past five years.

12. Did you use rounding or front –end estimation?

13. Why couldn't you use front –end estimation for this problem?

14. Carlos has been collecting change for the past few weeks. He has 5 nickels, 10 dimes, 6 quarters and four dollar bills. Write out each money amount.

15. Use rounding to estimate the sum of Carlos' money.

16. Use front –end estimation to estimate the sum of Carlos' money.

17. Which method gives you a more accurate estimate? Why?

18. Tina is working to buy presents for her family for the holidays. She has picked out a cd for her brother for \$14.69, a vase for her Mother at \$32.25 and a picture frame for her father at \$23.12. Use rounding to estimate the sum of Tina's purchases.

19. Use front –end estimation to find an estimate for the purchases.

20. Which estimate is more accurate?

21. Why?