CK-12

flexbook
next generation textbooks

Chapter 7: Equations, Inequalities and Functions

Michael McCallum

AUTHOR
Michael McCallum

To access a customizable version of this book, as well as other interactive content, visit www.ck12.org

CK-12 Foundation is a non-profit organization with a mission to reduce the cost of textbook materials for the K-12 market both in the U.S. and worldwide. Using an open-source, collaborative, and web-based compilation model, CK-12 pioneers and promotes the creation and distribution of high-quality, adaptive online textbooks that can be mixed, modified and printed (i.e., the FlexBook® textbooks).

Printed: November 27, 2017

flexbook
next generation textbooks

Contents

1 Equations, Inequalities and Functions 1

1.1 Addition and Subtraction Phrases as Equations . . . 2
1.2 Multiplication and Division Phrases as Equations . . . 6
1.3 Single Variable Equations from Verbal Models . . . 10
1.4 Simplify Sums or Differences of Single Variable Expressions . . . 15
1.5 Simplify Products or Quotients of Single Variable Expressions . . . 20
1.6 Simplify Variable Expressions Involving Multiple Operations . . . 27
1.7 Single Variable Addition Equations . . . 32
1.8 Single Variable Subtraction Equations . . . 37
1.9 Single Variable Multiplication Equations . . . 42
1.10 Single Variable Division Equation . . . 48
1.11 Two-Step Equations from Verbal Models . . . 53
1.12 Two-Step Equations and Properties of Equality . . . 57
1.13 Inequalities on a Number Line . . . 64
1.14 Two-Step Inequalities . . . 70
1.15 Domain and Range of a Function . . . 76
1.16 Input-Output Tables for Function Rules . . . 80
1.17 Graphs of Linear Equations . . . 88
1.18 Linear and Nonlinear Function Distinction . . . 97
1.19 Linear and Nonlinear Patterns of Change . . . 109
1.20 References . . . 118

CHAPTER 1 Equations, Inequalities and Functions

Chapter Outline

1.1 ADDITION AND SUBTRACTION PHRASES AS EQUATIONS
1.2 MULTIPLICATION AND DIVISION PHRASES AS EQUATIONS
1.3 SINGLE VARIABLE EQUATIONS FROM VERBAL MODELS
1.4 SIMPLIFY SUMS OR DIFFERENCES OF SINGLE VARIABLE EXPRESSIONS
1.5 SIMPLIFY PRODUCTS OR QUOTIENTS OF SINGLE VARIABLE EXPRESSIONS
1.6 SIMPLIFY VARIABLE EXPRESSIONS INVOLVING MULTIPLE OPERATIONS
1.7 SINGLE VARIABLE ADDITION EQUATIONS
1.8 SINGLE VARIABLE SUBTRACTION EQUATIONS
1.9 SINGLE VARIABLE MULTIPLICATION EQUATIONS
1.10 SINGLE VARIABLE DIVISION EQUATION
1.11 TWO-STEP EQUATIONS FROM VERBAL MODELS
1.12 TWO-STEP EQUATIONS AND PROPERTIES OF EQUALITY
1.13 INEQUALITIES ON A NUMBER LINE
1.14 TWO-STEP INEQUALITIES
1.15 DOMAIN AND RANGE OF A FUNCTION
1.16 INPUT-OUTPUT TABLES FOR FUNCTION RULES
1.17 GRAPHS OF LINEAR EQUATIONS
1.18 LINEAR AND NONLINEAR FUNCTION DISTINCTION
1.19 LINEAR AND NONLINEAR PATTERNS OF CHANGE
1.20 REFERENCES

Introduction

In this chapter, students will engage with the following concepts: writing expressions and equations, simplifying expressions, solving one-step equations, solving two-step equations, solving inequalities, identifying and understanding functions and graphing functions.

1.1 Addition and Subtraction Phrases as Equations

Learning Objectives

In this concept, you will learn how to write addition and subtraction phrases as single variable equations.

Sal leads an informal cycling team of six people. He wants to register them for a race, but doesn't know if there is enough space. The maximum number of allowed racers on the course is 138 cyclists. What is the maximum number of cyclists that can already be registered if the whole team can join the race? How can Sal write a single variable equation to represent this problem?

In this concept, you will learn how to write addition and subtraction phrases as single variable equations.

Writing Addition and Subtraction Phrases as Equations

An **expression** connects numbers and/or variables with operations, such as addition, subtraction, multiplication, and division. Notice that an expression does not have an equal sign. The value of the variable in each expression can change, and you can evaluate an expression using any value for the variable.

$$50-2 \qquad 4-a \qquad 12z \qquad \frac{4x}{3}$$

In the expressions above, $a, z,$ and x are variables.

An expression that includes one or more variables is called an **algebraic expression**. You can use algebraic expressions to represent words or phrases. To help you solve word problems, you need to translate words or phrases into operations, variables, or expressions. Let's start with addition and subtraction phrases. Take a look at this chart.

TABLE 1.1:

Addition Phrases		**Subtraction Phrases**	
1 **plus** a	$1+a$	4 **less** d	$4-d$
2 **and** b	$2+b$	6 **less than** g	$g-6$
4 **more than** c	$c+3$	h **fewer than** 7	$7-h$

The bolded words in the phrases above tell you if you should use addition or subtraction, and the order of the terms. Read the word problem carefully to figure out which operation makes the most sense.

Let's look at an example.

Abdul has \$5 more than Xavier has. Write an algebraic expression to show the number of dollars Abdul has.

The phrase is "\$5 more than Xavier." Use a number, an operation sign, or a variable to represent each part of that phrase.

Let x stand for the number of dollars Xavier has. "More than" means you are adding.

$$x+5$$

So, the expression $x+5$ represents the number of dollars Abdul has.

Let's look at another example.

Change "6 less than a number" into an algebraic expression.

Let "a number" be the variable x. "Less than" means you are subtracting. Be careful about the order, 6 will follow the variable.

$$x-6$$

The answer is $x-6$.

Here is one more example.

Lian is x inches shorter than Hannah. Hannah is 65 inches tall. Write an algebraic expression to show Lian's height in inches.

The phrase is "x inches shorter than Hannah." You also know that Hannah's height is 65 inches. "Shorter than" means you are subtracting. Be careful about the order, x will follow 65.

$$65-x$$

The answer is $65-x$.

In a word problem, the word "is" means equals. When you see the word "is" you can set the expression equal to something. This is an equation.

Examples

Example 1

Earlier, you were given a problem about Sal's biking team.

Sal needs to register his team of six people, but the maximum number of allowed racers on the course is 138 cyclists. Sal needs to write a single variable equation to figure out the maximum number of cyclists that can already be registered so that his team of six can join.

First, let x be the maximum number of cyclists that can be registered. Six more than that will give you the maximum of 138 cyclists. When you can use the phrase "more than" you can use addition.

The answer is $x+6=138$.

Example 2

Write an equation to represent the following phrase.

Four less than an unknown number is eighteen.

To figure this out, first let "an unknown number" be the variable x. Next, you know the operation is subtraction because of the key phrase "less than".

So you can write $x-4$ since the four is being taken away from the unknown number.

The word "is" means equals, and "eighteen" is 18.

$$x-4=18$$

The answer is $x-4=18$.

Example 3

Write an equation for the following phrase: a number plus five is ten.

Let x be "a number", "plus" means addition, and "is" means equals.

The answer is $x+5=10$.

Example 4

Write an equation for the following phrase: six more than a number is eighteen.

Let x be "a number", "more than" means addition, and "is" means equals.

The answer is $x+6=18$.

Example 5

Write an equation for the following phrase: fifteen less than a number is twenty.

Let x be "a number", "less than" means subtraction (but be careful about order), and "is" means equals.

The answer is $x-15=20$.

Review

Write an expression for each phrase.

1. 5 more than a number
2. A number plus six
3. 8 and a number
4. Seven less than a number
5. Eight take a way four
6. Nine more than a number

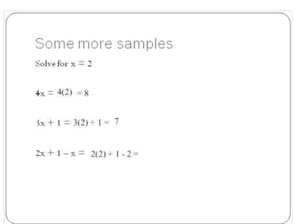
Write a simple equation for each phrase.

7. Five less than a number is ten.
8. Eight take away four is a number.
9. Five and a number is twelve.
10. Sixteen less than an unknown number is eighty.
11. Twenty and a number is fifty - five.
12. A number and fifteen is forty.
13. A number and twelve is sixty.
14. Fifteen less than a number is ninety.
15. Sixty less than a number is eighty.

Review (Answers)

To see the Review answers, open this PDF file and look for section 7.1.

Resources



MEDIA

Click image to the left or use the URL below.

URL: http://www.ck12.org/flx/render/embeddedobject/183574

1.2 Multiplication and Division Phrases as Equations

Learning Objectives

In this concept, you will learn how to write multiplication and division phrases as single variable equations.

There is a bunny population behind Ishmael's house. Ishmael is helping the local park ranger track the population numbers. At the start of July there are 127 females. By the end of the month, Ishmael and the ranger count 415 new bunnies. What is the average number of bunnies that each female had during that time? How can Ishmael write an equation that shows this situation?

In this concept, you will learn how to write multiplication and division phrases as single variable equations.

Writing Multiplication and Division Phrases as Equations

Just as you can write addition and subtraction expressions from words or phrases, you can also write multiplication and division expressions. You can use key words to help you with this. The more familiar you become with the key words that identify a multiplication or division expression, the better you will become at writing expressions.

Here are some phrases that can be translated into multiplication or division expressions.

TABLE 1.2:

Multiplication Phrases		Division Phrases	
9 **times** k	$9 \times k$ or $9k$	8 **divided into** n groups	$8 \div n$ or $\frac{8}{n}$
twice as much as m	$2 \times m$ or $2m$	q **shared equally** by 3 people	$q \div 3$ or $\frac{q}{3}$
		half of r	$r \div 2$ or $\frac{r}{2}$
		one-third of p	$p \div 3$ or $\frac{p}{3}$

Remember, these words are only a helpful guide. You should always think about which operation makes sense for a particular situation.

Let's look at an example.

Write an algebraic expression to represent the phrase "three times a number, t."

Use a number, an operation sign, or a variable to represent each part of the phrase.

First, consider the phrase and look for key terms.

three times a number, t

Next, rewrite the phrase using numbers and operations.

$3 \times t$

Then, simplify the expression.

$3t$

The phrase is represented by the expression $3t$.

Here is another example.

Mr. Warren separated 30 students into n equal groups. Write an algebraic expression to represent the number of students in each group.

First, consider the phrase and look for key terms.

30 students into n equal groups

Separating 30 students into n equal groups means dividing 30 students into n equal groups.

Next, write a division expression.

$\frac{30}{n}$

The phrase is represented by the expression $\frac{30}{n}$.

When an expression equals something it is an equation. If the number of students in each group needed to be five, for instance, you could write the following equation.

$n = 5$

Then, you could evaluate the expression $\frac{30}{n}$ using the given variable 5.

Examples

Example 1

Earlier, you were given a problem about Ishmael's local bunnies.

There are 127 females and 415 bunnies. Ishmael needs to write an equation that represents the average number of bunnies each female had.

First, let n be the average number of bunnies per female.

$127n$

This is the total number of bunnies in the population.

Then, because you know there are 415 bunnies, you can write an equation.

$127n = 415$

Example 2

Write a single variable equation for the following phrase.

Keith bought tickets to the movies. The tickets were \$8.50 each. Keith spent a total of \$34.00. How many tickets did Keith buy?

First, let t represent the number of tickets that Keith bought.

The cost of each ticket is \$8.50. The total price of the tickets will be the number of tickets, represented by t, times the price of each ticket.

Next, write an expression that represents this.

$8.5t$

Then, because you know the total cost of the tickets was \$34.00, write an equation.

$8.5t = 34.00$

Write a multiplication or division equation for the following phrases.

Example 3

Four times a number is eight.

Let x be "a number."

"Times" means multiplication and "is" means equals.

The answer is $4x = 8$.

Example 4

Sixteen candles divided into a number of piles is two candles in each pile. Let x be the number of piles.

"Divided into" means division but be careful about the order, and "is" means equals.

The answer is $\frac{16}{2} = 2$.

Example 5

The product of five and a number is fifteen.

Let x be "a number."

"Product" means you are multiplying, and "is" means equals.

The answer is $5x = 15$.

Review

Write an equation for each phrase.

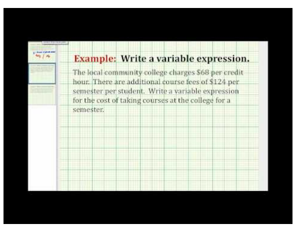
1. The product of four and a number is twelve.
2. Six times a number is thirty.
3. Twelve times a number is forty-eight.
4. Fourteen times a number is twenty-eight.
5. The product of five and a number is thirty.

6. Eight times a number is sixty-four.
7. Twenty divided by a number is four.
8. Eighty divided by a number is four.
9. Nineteen times an unknown number is ninety-five.
10. Thirteen times an unknown number is thirty-nine.
11. Twelve divided into groups is six.
12. An unknown number divided by two is eight.
13. An unknown number divided by seven is fourteen.
14. An unknown number times five is thirty-five.
15. An unknown number divided by twelve is twelve.

Review (Answers)

To see the Review answers, open this PDF file and look for section 7.2.

Resources



MEDIA
Click image to the left or use the URL below.
URL: http://www.ck12.org/flx/render/embeddedobject/182118

1.3 Single Variable Equations from Verbal Models

Learning Objectives

In this concept, you will learn to write single variable equations from verbal models.

Kelvin has twice as many chickens in his chicken coop as Murray has in his. If Kelvin has 60 chickens in his coop, write an equation to represent c, the number of chickens in Murray's chicken coop.

In this concept, you will learn to write single variable equations from verbal models.

Writing Single Variable Equations from Verbal Models

Changing a word problem into an equation can often help you in solving the problem. Carefully read the question to determine if the equation has addition, subtraction, multiplication, or division.

Here is an example.

Write "four times a number is twelve" as an equation.

First, let x be "a number."

Next, consider that the word "times" means you multiply.

$$4 \text{ times } x$$

$$4x$$

Then, consider that the word "is" means equals.

$$4x \text{ is } 12$$

The equation is $4x = 12$.

Here is another example.

Write "seven less than a number is fourteen" as an equation.

First, let x be "a number."

$$7 \text{ less than } x \text{ is } 14$$

Next, consider that "less than" means subtraction, but be careful about the order.

$$x - 7 \text{ is } 14$$

Then, consider that the word "is" means equals.

$$x - 7 = 14$$

The equation is $x - 7 = 14$.

Examples

Example 1

Earlier, you were given a problem about the chicken coops.

Kelvin has 60 chickens, which is twice as many chickens as Murray has. How can Kelvin write a single variable equation to represent his number of chickens?

First, simplify the language.

Kelvin has 60 chickens and this is 2 times c, the number of chickens Murray has.

Next, consider that "is" means equals, and "times" means multiplication.

$$\begin{aligned} 60 &= 2 \times c \\ 60 &= 2c \end{aligned}$$

The equation $60 = 2c$ represents the number of chickens in Murray's coop.

Example 2

Carrie made 3 liters of lemonade for a party. After the party, she had 0.5 liters of lemonade left. Write an equation to represent n, the number of liters of lemonade that her guests drank.

Use a number, an operation sign, a variable, or an equal sign to represent each part of the problem. Since the question tells you how many liters of lemonade were *left* after the party, this will be a subtraction equation.

Carrie started with 3 liters, and n is the number of liters that the guest drank. So, $3-n$ is how much lemonade there was after the party, 0.5 liters.

For this problem, it may help to write an equation in simplified words and then translate those words into an algebraic equation.

$$\begin{aligned}\text{(number of liters made)} - \text{(number of liters guests drank)} &= \text{(number of liters left)}\\ 3-n &= 0.5\end{aligned}$$

The equation is $3-n=0.5$.

Example 3

Write an equation for the following phrase: six and a number is twenty.

First, let x be "a number."

$$6 \text{ and } x \text{ is } 20$$

Next, consider that "and" means addition.

$$6+x \text{ is } 20$$

Then, consider that the word "is" means equals.

$$6+x=20$$

The equation is $6+x=20$.

Example 4

Write an equation for the following phrase: eighteen divided by a number is three.

First, let x be "a number."

$$18 \text{ divided by } x \text{ is } 3$$

Next, you know this involves division.

$$\frac{18}{x} \text{ is } 3$$

Then, consider that the word "is" means equals.

$$\frac{18}{x} = 3$$

The equation is $\frac{18}{x} = 3$.

Example 5

Write an equation for the following phrase: five times a number is twenty-five.

First, let y be "a number."

$$5 \times y \text{ is } 25$$

Next, consider that the word "times" means you multiply.

$$5y \text{ is } 25$$

Then, consider that the word "is" means equals.

$$5y = 25$$

The equation is $5y = 25$.

Review

Write an equation for each verbal model.

1. Ten times a number is thirty.
2. Five times a number is fifteen.
3. A number and seven is eleven.
4. A number divided by three is twelve.
5. A number and eighteen is thirty.
6. A number divided by twelve is fourteen.
7. Seven times a number is forty-nine.
8. A number divided by thirteen is seven.
9. Eight times a number is equal to sixty-four.

Write an algebraic expression for each situation below.

10. Arturo has 8 fewer stickers in his collection than Julissa has in hers. Let j represent the number of stickers in Julissa's collection. Write an expression to represent the number of stickers in Arturo's collection.
11. Let c represent the number of cookies on a plate. Three friends share all the cookies on the plate equally. Write an expression to represent the number of cookies each friend has after they are shared equally.
12. Carly is twice as old as her sister. Let s represent her sister's age in years. Write an expression to represent Carly's age in years.
13. The length of a rectangle is 3 inches longer than its width. Let w stand for the width in inches. Write an expression to represent the length in inches.

Write an algebraic equation for each word problem below.

14. The chorus teacher divides all the students in the chorus into 3 equal groups. Each of the groups has 6 students in it. Write an equation that could be used to represent n, the total number of students in the chorus.
15. Matt's dog weighs 30 pounds. His dog weighs 20 pounds more than his cat. Write an equation that could be used to represent c, the weight, in pounds, of Matt's cat.

Review (Answers)

To see the Review answers, open this PDF file and look for section 7.3.

1.4 Simplify Sums or Differences of Single Variable Expressions

Learning Objectives

In this concept, you will learn how to simplify single variable expressions.

Marc is heading to his local ice cream store. This particular store doesn't have a lot of flavor choices, but they have the BEST vanilla ice cream around. Marc has taken orders from several of his neighbors, too, and has written them all down so that he can keep track of who wants what.

The Johnsons - two vanilla ice cream cones

The Mumfords - three vanilla ice cream cones

Jill Stales - one vanilla ice cream cone

The Porters - three vanilla ice cream cones

How can Marc write this information as an algebraic expression and then simplify it?

In this concept, you will learn how to simplify single variable expressions.

Simplifying Sums or Differences of Single Variable Expressions

If an expression has only numbers, you can calculate its numerical value. However, if an expression includes variables, it is helpful to simplify the expression.

Look at this example.

Simplify the expression $6a+3a$.

When adding expressions with variables, it is important to remember that only like terms can be combined. For example, $6a$ and $3a$ are like terms because both terms include the variable a. So, you can combine them.

$$\begin{aligned}&6a+3a\\&9a\end{aligned}$$

Here is another example.

Simplify $6a+3$.

$6a$ and 3 are *not* like terms because only one term includes the variable a. So, you cannot combine them. The expression $6a+3$ cannot be simplified any further.

Here is another example.

Simplify $15d-2d$.

Since $15d$ and $2d$ both have the same variable, they are like terms. To find the difference, subtract the numerical parts of the terms the same way you would subtract any numbers.

$$15d-2d=13d$$

The answer is $13d$.

Here is an example using decimals.

Simplify $0.4x+1.3x$.

Since $0.4x$ and $1.3x$ both have the same variable, they are like terms. To find the sum, add the numerical parts of the terms the same way you would add any decimals.

$$0.4x+1.3x=1.7x$$

The answer is $1.7x$.

Examples

Example 1

Earlier, you were given a problem about Marc and all of his ice cream orders.

Marc needs to write an expression to simplify his order: two vanillas, three vanillas, one vanilla, and three vanillas.

Let v represent an ice cream cone. Then you can represent this situation as a single variable expression.

$$2v+3v+v+3v$$

Looking at this expression, you will see that the variables are all the same. Therefore, simply add the numerical part of each term.

$$2v+3v+v+3v=9v$$

The answer is $9v$.

Example 2

Simplify the expression.

$$5a+4a-2a+6a$$

To simplify this expression, follow the order of operations and combine like terms in order from left to right. Here is what the expression looks like after the first two terms have been combined.

$$9a-2a+6a$$

Next, perform the subtraction to get: $7a+6a$

Finally, add the terms.

$$7a+6a=13a$$

The answer is $13a$.

Simplify each sum or difference when possible.

Example 3

$$3a+12a$$

These are like terms, so add the numerical parts together.

The answer is $15a$.

Example 4

$$16x - 2x$$

These are like terms, so subtract the numerical parts.

The answer is $14x$.

Example 5

$$7y + 2x$$

These are not like terms.

The terms are not alike so you cannot combine them. The expression is in the simplest form already.

The answer is $7y + 2x$.

Review

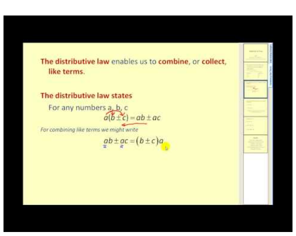
Simplify each sum or difference by combining like terms.

1. $6a + 7a$
2. $7x - 2x$
3. $6y + 12y$
4. $8a + 12a$
5. $12y - 7y$
6. $8a + 15a$
7. $13b - 9b$
8. $22x + 19x$
9. $45y - 12y$
10. $16a + 18a + 9a$
11. $14x - 6x + 2x$
12. $21a + 14a - 15a$
13. $33b + 13b + 8b$
14. $45x + 67x - 29x$
15. $92y + 6y - 54y$

Review (Answers)

To see the Review answers, open this PDF file and look for section 7.4.

Resources



MEDIA

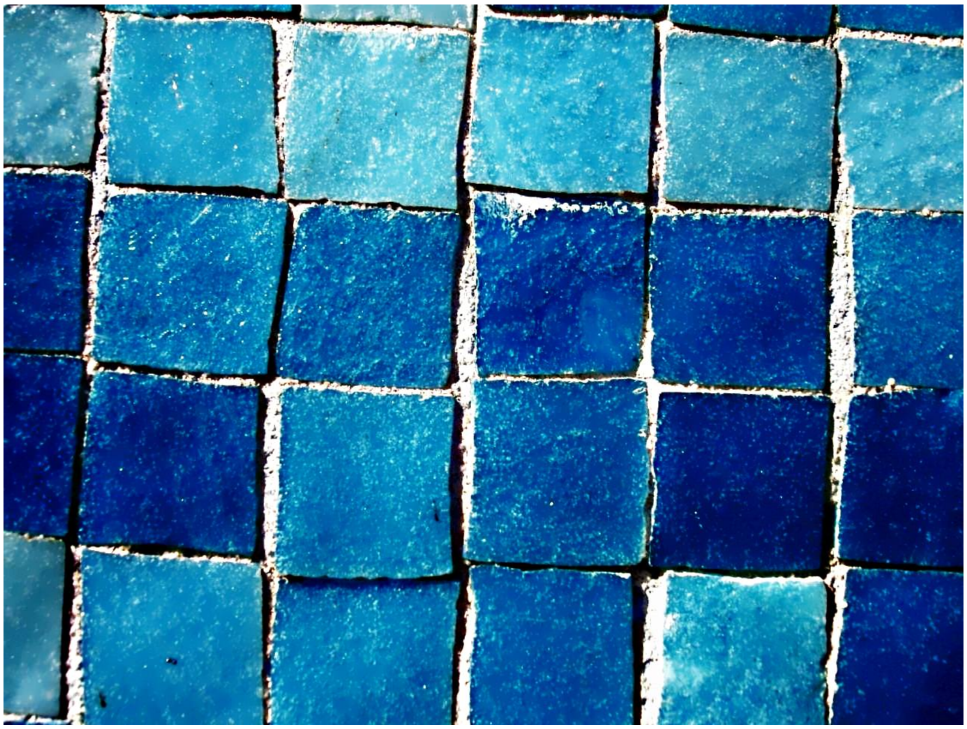
Click image to the left or use the URL below.

URL: http://www.ck12.org/flx/render/embeddedobject/183579

1.5 Simplify Products or Quotients of Single Variable Expressions

Learning Objectives

In this concept, you will learn to simplify products or quotients of single variable expressions.



Laine is helping his mother decorate the patio by putting tiles on a rectangular wall that is 20 tiles wide and 32 tiles high. If each tile is a square with side *s,* then the total area is $20s \times 30s$. How can Laine figure out the total area to be tiled in simplified form?

In this concept, you will learn to simplify products or quotients of single variable expressions.

Simplifying Products or Quotients of Single Variable Expressions

With expressions, **terms** are separated from each other by addition or subtraction, while **factors** are separated by multiplication or division. For example, the expression $3ab - 7ba$ is composed of two terms, $3ab$ and $7ba$, and each of those terms is composed of three factors.

When adding or subtracting terms in an expression, you can only combine **like terms**, which are composed of only the same variables. However, you can multiply or divide terms whether they are like terms or not.

For example, $3ab$ and $7ba$ are like terms - both terms include only the variables of a and $b,$ regardless of the order in which they appear in each term. However, $3a$ and $7b$ are *not* like terms, since one contains the variable $a,$ and the other contains the variable b. Since $3a$ and $7b$ are not like terms, they can't be added together:

$$3a + 7b = 3a + 7b$$

There is nothing you can do to simplify the expression.

However, $3a$ and $7b$ *can* be multiplied by each other:

$$3a \times 7b = 21ab$$

Clearly, $21ab$ is simpler than $3a \times 7b$.

Two rules will help you multiply expressions that contain variables. The **Commutative Property of Multiplication** states that two terms can be multiplied in any order. **The Associative Property of Multiplication** states that the grouping of terms does not change your answer.

It is also helpful to remember that when multiplying like variables together, you add the exponents.

For example, remember that x is the same as x^1:

$$\begin{aligned} x(x) &= x^2 \\ x(x)(x) &= x^3 \\ x^2(x) &= x^3 \end{aligned}$$

Look at the following example.

Simplify $6a(3a)$.

First, multiply the number parts.

$$6 \times 3 = 18$$

Next, multiply the variables.

$$a \cdot a = a^2$$

The answer is $18a^2$.

Here is another example.

Simplify $5 \times (8y)$.

These are not like terms, since they contain different variables, but they can still be multiplied.

First, multiply the numbers.

$$5 \times 8 = 40$$

Next, multiply the variables.

$$x \cdot y = xy$$

The answer is $40xy$.

Here is one more example.

Find the product $4z \times \frac{1}{2}$.

$4z$ and $\frac{1}{2}$ are not like terms, however, you can multiply terms even if they are not like terms.

Use the commutative and associative properties to rearrange the factors to make it easier to see how they can be multiplied.

According to the commutative property, $z\left(\frac{1}{2}\right) = \frac{1}{2}(z)$.

$$4z \times \frac{1}{2} = 4 \times \frac{1}{2}(z)$$

According to the associative property, the grouping of the factors does not change the answer. Group the factors so that the numbers are multiplied first.

$$4 \times \frac{1}{2}(z) = 4 \times \frac{1}{2} \times z = \left(4 \times \frac{1}{2}\right) \times z$$

Now, multiply.

$$\left(4 \times \frac{1}{2}\right) \times z = (2) \times z = 2z$$

The answer is $2z$.

Here is an example using division.

Find the quotient $42c \div 7$.

First, rewrite the problem like this $\frac{42c}{7}$.

Then separate out the numbers and variables like this.

$$\frac{42c}{7} = \frac{42 \cdot c}{7} = \frac{42}{7} \cdot c$$

Now, divide 42 by 7 to find the quotient.

$$\frac{42}{7} \cdot c = 6 \cdot c = 6c$$

The answer is $6c$.

Examples

Example 1

Earlier, you were given a problem about Laine, who is decorating the patio by gluing tiles on a rectangular wall 20 tiles tall (long) and 30 tiles wide.

Each tile is a square with side s. Since the area of a rectangle is length times width, the total area is $20s \times 30s$.

Laine needs to know what the total area is to be tiled in simplified form. So he will need to multiply 20s by 30s.

First, multiply the numbers.

$$20 \times 30 = 600$$

Next, multiply the variables.

$$s \times s = s^2$$

The answer is $600s^2$.

Example 2

Find the quotient of $50g \div 10g$.

First, rewrite the problem.

$$\frac{50g}{10g}$$

Then, separate the factors.

$$\frac{50 \cdot g}{10 \cdot g}$$

Next, reduce and cancel.

$$\frac{\cancel{50}\,5 \cdot \cancel{g}}{\cancel{10}\,1 \cdot \cancel{g}} = \frac{5}{1} = 5$$

The answer is 5.

Example 3

Use the commutative and associative properties of multiplication to simplify $6a(9a)$.

First, apply the associative property to separate the a and the 9.

$$6a(9a) = 6(a9)a$$

Next, apply the commutative property to put the numbers and variables next to each other.

$$6(a9)a = 6(9a)a$$

Then, apply the associative property again to group the similar factors.

$$6(9a)a = (6 \cdot 9)(a \cdot a)$$

Finally, multiply the similar factors.

$$(6 \cdot 9)(a \cdot a) = 54a^2$$

The answer is $54a^2$.

Example 4

Use the commutative and associative properties of multiplication to simplify $15b \div 5b$.

First, rewrite the problem in vertical format.

$$\frac{15b}{5b}$$

Next, separate the factors.

$$\frac{15 \cdot b}{5 \cdot b}$$

Then, identify and cancel similar factors.

$$\frac{\cancel{5} \cdot 3 \cdot \cancel{b}}{\cancel{5}1 \cdot \cancel{b}}$$

Finally, simplify to get the answer.

$$\frac{3}{1} = 3$$

The answer is 3.

Example 5

Simplify $\frac{20c}{4}$.

First, separate the factors.

$$\frac{20 \cdot c}{4}$$

Next, identify and cancel similar factors.

$$\frac{5 \cdot 4 \cdot c}{4}$$

Finally, simplify to get the answer.

$$\frac{5c}{1} = 5c$$

The answer is $5c$.

Review

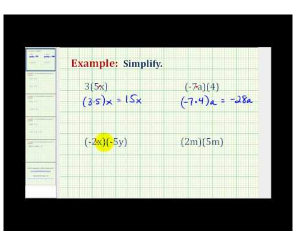
Simplify each product or quotient.

1. $6a(4a)$
2. $9x(2)$
3. $14y(2y)$
4. $16a(a)$
5. $22x(2x)$
6. $18b(2)$
7. $21a \div 7$
8. $22b \div 2b$
9. $25x \div x$
10. $45a \div 5a$
11. $15x \div 3x$
12. $18y \div 9$
13. $22y \div 11y$
14. $\frac{15x}{3y}$
15. $\frac{82x}{2x}$

Review (Answers)

To see the Review answers, open this PDF file and look for section 7.5.

Resources



MEDIA

Click image to the left or use the URL below.

URL: http://www.ck12.org/flx/render/embeddedobject/183580

1.6 Simplify Variable Expressions Involving Multiple Operations

Learning Objectives

In this concept, you will learn to simplify variable expressions involving multiple operations.

A farmer, Kelly, has two lovely plots of rectangular land to grow some vegetables. Her friend will help her plant in the spring. One plot of land is $8a$ by $6a$ and the other plot of land is $3a$ by $4a$. The two plots of land are going to be combined so they can grow more vegetables. How can Kelly find the total area for both plots of farmable land?

In this concept, you will learn to simplify variable expressions involving multiple operations.

Simplifying Variable Expressions Involving Multiple Operations

Sometimes, you may need to simplify algebraic expressions that involve more than one operation. Use what you know about simplifying sums, differences, products, or quotients of algebraic expressions to help you do this.

When evaluating expressions, it is important to keep in mind the **order of operations**, which is

- First, do the computation inside parentheses.
- Second, evaluate any exponents.
- Third, multiply and divide in order from left to right.
- Finally, add and subtract in order from left to right.

Now let's look at an example.

Simplify this expression $7n+8n\cdot 3$

First, simplify according to the order of operations. According to the order of operations, you should multiply first.

$$7n + 8n \cdot 3 = 7n + 24n.$$

Next, add like terms.

$$7n + 24n = 31n$$

The answer is $31n$.

Here is another example.

Simplify the expression $10p - 7p + 8p \div 2p$.

First, follow the order of operations and rewrite the division as a fraction.

$$\frac{8p}{2p}$$

Next, simplify the fraction, assuming p is not equal to zero.

$$\frac{8p}{2p} = \frac{8}{2} \times \frac{p}{p} = 4 \times 1 = 4$$

Next, rewrite the equation.

$$10p - 7p + \frac{8p}{2p} = 10p - 7p + 4$$

Simplify, by combing like terms.

$$10p - 7p + 4 = 3p + 4$$

The answer is $3p + 4$.

Examples

Example 1

Earlier, you were given a problem about Kelly and her two plots of land.

One is $8a$ by $6a$ and the other is $3a$ by $4a$.

The plots of land need to be combined to find the total area of farmable land.

First, consider the equation for the area of a rectangle.

$$\text{Area of a rectangle} = \text{length} \times \text{width}$$

Next, calculate the area of the first rectangular plot of land.

$$8a \times 6a = 48a^2$$

Then, calculate the area of the second rectangular plot of land.

$$3a \times 4a = 12a^2$$

Next, write and expression for the total area of the two plots of land.

$$48a^2 + 12a^2$$

Finally combine like terms.

$$48a^2 + 12a^2 = 60a^2$$

The answer is $60a^2$.

Example 2

Samera has twice as many pets as Amit has. Kyra has 4 times as many pets as Amit has. Let a represent the number of pets Amit has.

a. Write an expression to represent the number of pets Samera has.
b. Write an expression to represent the number of pets Kyra has.
c. Write an expression to represent the number of pets Samera and Kyra have all together.

First, answer part *a*.

Samera has twice as many pets as Amit. Since Amit has a pets, Samera has $2a$ pets.

Next, answer part *b*.

Kyra has 4 times as many pets as Amit has. Since Amit has a pets, Kyra has $4a$ pets.

Finally, answer part *c*.

To find the number of pets Samera and Kyra have "all together," write an addition expression.

$$2a + 4a$$

Finally, combine like terms.

$$2a + 4a = 6a$$

The answer is $6a$.

Simplify each expression.

Example 3

$$4a+9a-7$$

Combine the like terms $4a$ and $9a$.

The answer is $13a-7$.

Example 4

$$\frac{14x}{2}+9x$$

First, follow the order of operations and do the division first.

$$\frac{14x}{2}+9x=7x+9x$$

Next, combine like terms.

$$7x+9x=16x$$

The answer is $16x$.

Example 5

$$6b-2b+5b-8$$

Follow the order of operations and perform the addition and subtraction from left to right.

$$6b-2b+5b-8=9b-8$$

The answer is $9b-8$.

Review

Simplify each expression involving multiple operations.

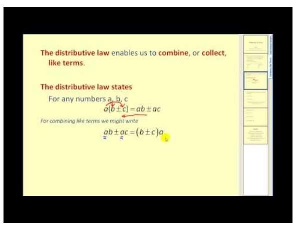
1. $6a+4a-2b$
2. $16b-4b\cdot 2$
3. $22a\div 2+14a$
4. $19x-5x\cdot 2$
5. $16y-12y\div 2$
6. $16a-4a-12b$
7. $26a+14a+12b+2b$

8. $36a + 4a - 2b + 5b$
9. $18a + 4a + 12y$
10. $46a + 34a - 12b + 14b$
11. $16y + 4y - 2x$
12. $6x + 4x + 2x + 4y - 19z$
13. $26y - 12y \div 2$
14. $36y - 12y \div 12$
15. $46y + 12y \div 2$

Review (Answers)

To see the Review answers, open this PDF file and look for section 7.6.

Resources



MEDIA

Click image to the left or use the URL below.

URL: http://www.ck12.org/flx/render/embeddedobject/183579

1.7 Single Variable Addition Equations

Learning Objectives

In this concept, you will learn to solve single variable addition equations.

The language club is fundraising for a trip to study art and architecture in Paris next summer. They have raised \$3,500 from helping the community and \$4,800 from various donors. They need a total of \$12,300 to subsidize the educational trip. Can you write an equation to solve for how much more they need to raise, and then find out that amount?

In this concept, you will learn to solve single variable addition equations.

Solving Single Variable Addition Equations

A **variable** is used to represent a number, quantity, or expression.

For example, in the algebraic equation below, the variable x represents one possible number.

$$x+3=5$$

To find out what number x represents, ask yourself, "What number, when added to 3, equals 5?"

$2+3=5$, so x must be equal to 2.

When solving more complex equations, such as $x+34=72$, it is important to be more systematic and strategic.

To solve an equation you should work to isolate the variable. Isolating the variable means getting the variable by itself on one side of the equal $(=)$ sign.

One way to isolate the variable is to use an **inverse operation**, that is, the 'opposite' operation. For example, addition is the inverse of subtraction, subtraction is the inverse of addition, multiplication is the inverse of division, and division is the inverse of multiplication.

To solve an equation in which a variable is added to a number, you can use the inverse of addition–subtraction.

To do this you need to use the **Subtraction Property of Equality**, which states: if $a = b$, then $a - c = b - c$.

This means that if you subtract a number, c, from one side of an equation, you must subtract that same number, c, from the other side, too, to keep the values on both sides equal.

Let's look at an example.

Solve for x.

$x + 34 = 72$

Use the subtraction property of equality to subtract 34 from both sides of the equation. This will isolate the variable x.

$$\begin{aligned} x + 34 &= 72 \\ x + 34 - 34 &= 72 - 34 \\ x + 0 &= 38 \\ x &= 38 \end{aligned}$$

The answer is $x = 38$.

Here is another example.

Solve for b.

$1.5 + b = 3.5$

In the equation, 1.5 is added to b. So, use the subtraction property of equality and subtract 1.5 from both sides of the equation to solve for b.

$$\begin{aligned} 1.5 + b &= 3.5 \\ 1.5 - 1.5 + b &= 3.5 - 1.5 \\ 0 + b &= 2.0 \\ b &= 2 \end{aligned}$$

The answer is $b = 2$.

Examples

Example 1

Earlier, you were given a problem about the language club, who is going to study art and architecture in Paris.

They have raised \$3,500 from one source and \$4,800 from another. However, they need a total of \$12,300 for the educational trip. Write an equation to solve for how much more they need to raise.

First, let x be how much money the students still need to raise.

Next, you need to translate the language into a mathematical equation. Add up all of the funding plus what they still need to raise, x, and set that equal to the total amount needed.

$$3,500+4,800+x=12,300$$

Next, add the numbers together.

$$8,300+x=12,300$$

Now, use the subtraction property of equality to subtract 8,300 from both sides of the equation. This isolates the variable x.

$$\begin{aligned}8,300-8,300+x &= 12,300-8,300\\ x &= 4,000\end{aligned}$$

The answer is the students still need to raise $4,000.

Example 2

The number of gray tiles in a bag is 4 more than the number of blue tiles in the bag. There are 11 gray tiles in the bag.

Write an equation to represent b, the number of blue tiles in the bag, and then find the value of b.

First, translate the language into a mathematical equation. “Is” means equals, b is the number of blue tiles, and there are 11 gray tiles.

$$\begin{array}{cccccc}\text{The } \underline{\text{number of gray tiles}} \ldots & \underline{\text{is 4 more than}} & & \text{the } \underline{\text{number of blue tiles}} \ldots\\ \downarrow & \downarrow & \downarrow & \downarrow\\ 11 & =4 & + & b\end{array}$$

So the equation is $11=4+b$.

Solve the equation to find the number of blue tiles in the bag. Use the subtraction property of equality, and subtract 4 from each side of the equation. This isolates the variable.

$$\begin{aligned}11 &= 4+b\\ 11-4 &= 4-4+b\\ 7 &= 0+b\\ 7 &= b\end{aligned}$$

The answer is there are 7 blue tiles in the bag.

Solve each addition equation for the missing variable.

Example 3

$x+36=90$

Use the subtraction property of equality and subtract 36 from both sides of the equation. This isolates the variable x.

$$\begin{aligned} x+36 &= 90 \\ x+36-36 &= 90-36 \\ x+0 &= 54 \\ x &= 54 \end{aligned}$$

The answer is $x=54$.

Example 4

$x+27=35$

Use the subtraction property of equality and subtract 27 from both sides of the equation. This isolates the variable x.

$$\begin{aligned} x+27 &= 35 \\ x+27-27 &= 35-27 \\ x &= 8 \end{aligned}$$

The answer is $x=8$.

Example 5

$y+1.7=6.5$

Use the subtraction property of equality and subtract 1.7 from both sides of the equation. This isolates the variable y.

$$\begin{aligned} y+1.7 &= 6.5 \\ y+1.7-1.7 &= 6.5-1.7 \\ y &= 4.8 \end{aligned}$$

The answer is $y=4.8$.

Review

Solve each single-variable addition equation.

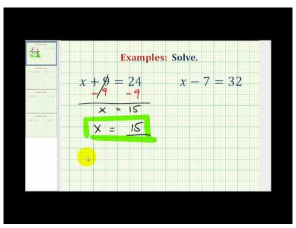
1. $x+7=14$
2. $y+17=34$
3. $a+27=34$
4. $x+30=47$
5. $x+45=53$
6. $x+18=24$

7. $a+38=74$
8. $b+45=80$
9. $c+54=75$
10. $y+197=423$
11. $y+297=523$
12. $y+397=603$
13. $y+97=405$
14. $y+94=102$
15. $y+87=323$

Review (Answers)

To see the Review answers, open this PDF file and look for section 7.7.

Resources



MEDIA

Click image to the left or use the URL below.

URL: http://www.ck12.org/flx/render/embeddedobject/182121

1.8 Single Variable Subtraction Equations

Learning Objectives

In this concept, you will learn to solve single variable subtraction equations.

The baking club realized they only have 2.7 kg of flour left from their original order. The only person to use the flour since they bought it was Sal who used .9 kg of flour to make cakes. Can you write an equation to solve for how much flour they had before Sal took some, and then solve that equation?

In this concept, you will learn to solve single variable subtraction equations.

Solving Single Variable Subtraction Equations

To solve an equation in which a number is subtracted from a variable, you can use addition to isolate the variable.

You can add the same number to both sides of the equation because of the **Addition Property of Equality**, which states: if $a = b$, then $a + c = b + c$.

This means that if you add a number, c, to one side of an equation, you must add that same number, c, to the other side, in order to keep both sides of the equation equal.

Let's look at an example.

Solve for a.

$$a - 15 = 18$$

In the equation, 15 is subtracted from a. So, you use the addition property of equality to add 15 to both sides of the equation. This will isolate the variable, a.

$$\begin{aligned} a-15 &= 18 \\ a-15+15 &= 18+15 \\ a+-15+15 &= 33 \\ a+0 &= 33 \\ a &= 33 \end{aligned}$$

Notice that you can rewrite (-15) as $(+-15)$. This is a very useful strategy in solving equations. You can rewrite subtraction as adding a negative number.

The answer is $a = 33$.

Here is another example.

Solve for k.

$$k - \frac{1}{3} = \frac{2}{3}$$

In the equation, $\frac{1}{3}$ is subtracted from k. So, you use the addition property of equality, and add $\frac{1}{3}$ to both sides of the equation. This isolates the variable k.

First, add $\frac{1}{3}$ to both sides of the equation.

$$\begin{aligned} k-\tfrac{1}{3} &= \tfrac{2}{3} \\ k-\tfrac{1}{3}+\tfrac{1}{3} &= \tfrac{2}{3}+\tfrac{1}{3} \end{aligned}$$

Since the fractions have the same denominators you can add them together.

$$\begin{aligned} k-\tfrac{1}{3}+\tfrac{1}{3} &= \tfrac{2}{3}+\tfrac{1}{3} \\ k &= \tfrac{3}{3} \\ k &= 1 \end{aligned}$$

The answer is $k = 1$.

Examples

Example 1

Earlier, you were given a problem about the baking club and Sal, who used their flour to make cakes.

They need to know how much flour they originally had so they can pay their supplier.

The club started with some flour, but Sal took .9 kg of this flour to make cake. They had 2.7 kg left. You have to find how much they had to begin with.

First, you need to write an equation that represents this information. Let x, represent the amount of flour the group originally had. This amount, minus the .9 kg Sal took equals 2.7 kg.

$$x - .9 = 2.7$$

Use the addition property of equality and add .9 to both sides of the equation and solve.

$$\begin{aligned} x - .9 + .9 &= 2.7 + .9 \\ x &= 3.6 \end{aligned}$$

The answer is they originally had 3.6 kg of flour.

Example 2

Harry earned \$19.50 this week. That is \$6.50 less than he earned last week.

Write an equation to represent m, the amount of money, in dollars, that he earned last week. Then solve for m.

Let m be the amount of money Harry earned last week. Then you can write an equation.

$$19.50 = m - 6.50.$$

Next, solve the equation. Use the addition property of equality to add 6.50 to each side of the equation.

$$\begin{aligned} 19.50 &= m - 6.50 \\ 19.50 + 6.50 &= m - 6.50 + 6.50 \\ 26.00 &= m + (-6.50 + 6.50) \\ 26 &= m + 0 \\ 26 &= m \end{aligned}$$

The answer is Harry earned \$26.00 last week.

Solve each equation.

Example 3

Solve for x.

$$x - 44 = 22$$

Use the addition property of equality and add 44 to both sides of the equation.

$$\begin{aligned} x - 44 &= 22 \\ x - 44 + 44 &= 22 + 44 \\ x &= 66 \end{aligned}$$

The answer is $x = 66$.

Example 4

Solve for x.

$$x-1.3=5.6$$

Use the addition property of equality and add 1.3 to both sides of the equation.

$$\begin{aligned} x-1.3 &= 5.6 \\ x-1.3+1.3 &= 5.6+1.3 \\ x &= 6.9 \end{aligned}$$

The answer is $x=6.9$.

Example 5

Solve for y.

$$y-\frac{1}{4}=\frac{1}{2}$$

Use the addition property of equality and add $\frac{1}{4}$ to both sides of the equation.

$$\begin{aligned} y-\tfrac{1}{4} &= \tfrac{1}{2} \\ y-\tfrac{1}{4}+\tfrac{1}{4} &= \tfrac{1}{2}+\tfrac{1}{4} \\ y &= \tfrac{1}{2}+\tfrac{1}{4} \end{aligned}$$

To add $\frac{1}{2}+\frac{1}{4}$ you need common denominators. Write $\frac{1}{2}$ as the equivalent fraction $\frac{2}{4}$ and then add.

$$\begin{aligned} y &= \tfrac{2}{4}+\tfrac{1}{4} \\ y &= \tfrac{3}{4} \end{aligned}$$

The answer is $y=\frac{3}{4}$.

Review

Solve each single-variable subtraction equation.

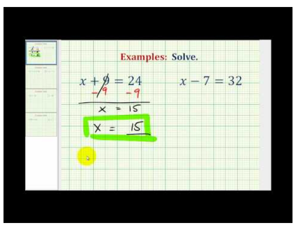
1. $x-8=9$
2. $x-18=29$
3. $a-9=29$
4. $a-4=30$
5. $b-14=27$
6. $b-13=50$
7. $y-23=57$

8. $y - 15 = 27$
9. $x - 9 = 32$
10. $c - 19 = 32$
11. $x - 1.9 = 3.2$
12. $y - 2.9 = 4.5$
13. $c - 6.7 = 8.9$
14. $c - 1.23 = 3.54$
15. $c - 5.67 = 8.97$

Review (Answers)

To see the Review answers, open this PDF file and look for section 7.8.

Resources



MEDIA

Click image to the left or use the URL below.

URL: http://www.ck12.org/flx/render/embeddedobject/182121

1.9 Single Variable Multiplication Equations

Learning Objectives

In this concept, you will learn to solve single variable multiplication equations.

Mr. Ricky's Biology class is going to the botanical gardens to study the different kinds of flowers and plants that are there. The students have raised 68 dollars for tickets so far. The student rate for each ticket is 5 dollars. Mr. Ricky asked the students to figure out how many tickets they can buy. How many tickets can they afford with 68 dollars?

In this concept, you will learn to solve equations with one variable and involve multiplication.

Solving Single Variable Multiplication Equations

In the algebraic equation below, the variable z represents a number.

$$z \times 2 = 8$$

What number does z represent?

You can find out by asking yourself, "What number, when multiplied by 2, equals 8?"

Since $4 \times 2 = 8$, z must be equal to 4.

Sometimes, particularly with larger numbers, it can be more difficult to just 'know' the answer. Consider $z \times 7 = 105$, since you may not know right away what number, multiplied by 7, equals 105, you should use another strategy for solving the equation.

To solve a more challenging equation in which a variable is multiplied by a number, you can use the inverse operation of multiplication-division. To isolate the variable, divide both sides of the equation by the number the variable is multiplied by.

You can divide both sides of the equation by the same number and not change the equality because of the **Division Property of Equality**, which says that two things that are the same will still be the same if they are both divided by an equal amount. In math language, this looks like:

If $a = b$ and $c \neq 0$, then $\frac{a}{c} = \frac{b}{c}$.

In other words, if you divide one side of an equation by a nonzero number, c, you must divide the other side of the equation by that same number, c, to keep the values on both sides equal.

Let's look at an example.

Solve for z.

$$z \times 7 = 105$$

In this equation, z is multiplied by 7. So, to isolate the variable z, you can divide both sides of the equation by 7.

First, using the division property of equality, divide both sides of the equation by 7.

$$\begin{aligned} z \times 7 &= 105 \\ \frac{z \times 7}{7} &= \frac{105}{7} \end{aligned}$$

Next, separate the fraction $\frac{z \times 7}{7}$, and simplify.

$$\begin{aligned} z \times \frac{7}{7} &= 15 \\ z \times 1 &= 15 \\ z &= 15 \end{aligned}$$

The answer is $z = 15$.

Here is another example.

Solve for r: -8$r = 128$

In this equation, -8 is multiplied by r. So, using the division property of equality, you can divide both sides of the equation by -8 to solve for r.

First, divide both sides of the equation by -8.

$$\begin{aligned} -8r &= 128 \\ \frac{-8r}{-8} &= \frac{128}{-8} \end{aligned}$$

Next, separate the fraction $\frac{-8r}{-8}$, and simplify.

$$\begin{aligned} \frac{-8r}{-8} &= \frac{128}{-8} \\ 1r &= -16 \\ r &= -16 \end{aligned}$$

The answer is $r =$ -16.

Examples

Example 1

Earlier, you were given a problem about Mr. Ricky's Biology class.

The students have raised 68 dollars for their trip and are wondering how many 5 dollar tickets they can buy.

First, write an equation to represent this information. Let d, be the number of tickets they can buy. You can say that d times the price of each ticket, 5 dollars, is 68 dollars.

$$5d = 68$$

Next, use the division property of equality and divide both sides of the equation by 5.

$$\frac{5d}{5} = \frac{68}{5}$$

Then, separate the fraction $\frac{5d}{5}$ and simplify.

$$\begin{aligned} \tfrac{5}{5}d &= 13.6 \\ 1d &= 13.6 \\ d &= 13.6 \end{aligned}$$

Next, interpret the result.

The students can buy 13.6 tickets. You can't buy .6 of a ticket. So, the students can only buy 13 tickets with 3 dollars left over.

The answer is the students can buy 13 tickets.

Example 2

Sarvenaz earns $8 for each hour she works. She earned a total of $168 last week.

a. Write an equation to represent h, the number of hours she worked last week.
b. Determine how many hours Sarvenaz worked last week.

First, complete part a.

Let h be the number of hours Sarvenaz worked. She earns $8 for each hour she works, so you multiply the number of hours she worked by $8 to find the total amount she earned. Write a multiplication equation.

$$8h = 168$$

Next, work on part b.

Solve the equation $8h = 168$ to find h, the number of hours she worked last week.

First, use the division property of equality to divide both sides of the equations by 8.

$$\begin{aligned} 8h &= 168 \\ \frac{8h}{8} &= \frac{168}{8} \end{aligned}$$

Next, separate the fraction $\frac{8h}{8}$ and simplify.

$$\begin{aligned} \frac{8}{8}h &= 21 \\ 1h &= 21 \\ h &= 21 \end{aligned}$$

The answer is Sarvenaz works 21 hours last week.

Solve each equation.

Example 3

$$-4x = 12$$

First, use the division property of equality, and divide both sides of the equation by -4.

$$\begin{aligned} -4x &= 12 \\ \frac{-4x}{-4} &= \frac{12}{-4} \end{aligned}$$

Next, separate the fraction $\frac{-4x}{-4}$ and simplify.

$$\begin{aligned} \frac{-4}{-4}x &= -3 \\ 1x &= -3 \\ x &= -3 \end{aligned}$$

The answer is $x = -3$.

Example 4

$$8a = 64$$

First, use the division property of equality and divide both sides of the equation by 8.

$$\begin{aligned} 8a &= 64 \\ \frac{8a}{8} &= \frac{64}{8} \end{aligned}$$

Next, separate the fraction $\frac{8a}{8}$ and simplify.

$$\begin{aligned} \frac{8}{8}a &= 8 \\ 1a &= 8 \\ a &= 8 \end{aligned}$$

The answer is $a = 8$.

Example 5

$$9b = 81$$

First, use the division property of equality and divide both sides of the equation by 9.

$$\begin{aligned} 9b &= 81 \\ \frac{9b}{9} &= \frac{81}{9} \end{aligned}$$

Next, separate the fraction $\frac{9b}{9}$ and simplify.

$$\begin{aligned} \frac{9}{9}b &= 9 \\ 1b &= 9 \\ b &= 9 \end{aligned}$$

The answer is $b = 9$.

Review

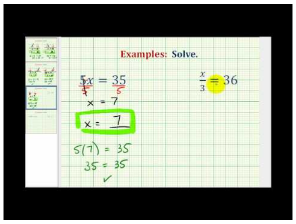
Solve each single variable multiplication equation for the missing value.

1. $4x = 16$
2. $6x = 72$
3. $-6x = 72$
4. $-3y = 24$
5. $-3y = -24$
6. $-5x = -45$
7. $-1.4x = 2.8$
8. $3.5a = 7$
9. $7a = -49$
10. $14b = -42$
11. $24b = -48$
12. $-24b = -48$
13. $34b = -102$
14. $84x = 252$
15. $-84x = -252$

Review (Answers)

To see the Review answers, open this PDF file and look for section 7.9.

Resources



MEDIA

Click image to the left or use the URL below.

URL: http://www.ck12.org/flx/render/embeddedobject/182123

1.10 Single Variable Division Equation

Learning Objectives

In this concept, you will learn to solve single variable division equations.

The incoming class at tennis camp is large this year. This is a special program that will have 6 people in each class. However, there is a maximum number of 42 classes. The incoming people need to be divided into groups of 6, so that the number of groups is 42. What is the maximum capacity, c, of the incoming class? How can you write an equation for c, and then solve this equation?

In this concept, you will learn to solve single variable division equations.

Solving Single Variable Division Equations

To solve an equation in which a variable is divided by a number, you use the inverse of division, multiplication, to isolate the variable and solve the equation.

You can multiply both sides of an equation by a number because of the **Multiplication Property of Equality**, which states:

if $a = b$, then $a \times c = b \times c$.

This means that if you multiply one side of an equation by a number c, you must multiply the other side of the equation by that same number c, to keep the values on both sides of the equation equal.

Here is an example.

Solve the equation for k.

$\frac{k}{-4} = 12$

First, use the multiplication property of equality, and multiply both sides of the equation by -4 to isolate the variable k.

$$\begin{aligned} \frac{k}{-4} &= 12 \\ -4 \times \frac{k}{-4} &= -4 \times 12 \\ \frac{-4k}{-4} &= -48 \end{aligned}$$

Next, separate the fraction and simplify.

$$\begin{aligned} \frac{-4}{-4}k &= -48 \\ 1k &= -48 \\ k &= -48 \end{aligned}$$

The answer is $k = -48$.

Here is another example.

Solve for n in the equation $\frac{n}{1.5} = 10$.

First, use the multiplication property of equality to multiply both sides of the equation by 1.5.

$$1.5 \times \frac{n}{1.5} = 10 \times 1.5$$

Next, since $1.5\frac{n}{1.5} = \frac{1.5}{1} \times \frac{n}{1.5}$, you can rewrite that multiplication as one fraction.

$$\frac{1.5n}{1.5} = 10 \times 1.5$$

Next, you separate the fraction and simplify.

$$\begin{aligned} \frac{1.5}{1.5}n &= 15 \\ 1n &= 15 \\ n &= 15 \end{aligned}$$

The answer is $n = 15$.

Examples

Example 1

Earlier, you were given a problem about the tennis camp.

The incoming students need to be divided into groups of 6, but there can only be 42 classes in total. Can you write a division equation, where c, is the maximum number of people in the incoming class so that there are 6 people in each group, and then solve it?

First, translate the language into an equation. Let c, be the maximum number of people in the incoming class. This number, divided by 6, should equal 42 classes.

$$\frac{c}{6} = 42$$

Next, use the multiplication property of equality and multiply both sides of the equation by 6.

$$6 \times \frac{c}{6} = 6 \times 42$$

Then, re-write the multiplication by a fraction and simplify.

$$\begin{aligned} \tfrac{6}{1} \times \tfrac{c}{6} &= 252 \\ \tfrac{6}{6} \times \tfrac{c}{1} &= 252 \\ 1c &= 252 \\ c &= 252 \end{aligned}$$

The answer is that there can be a maximum number of 252 students in the incoming class.

Example 2

Three friends evenly split the total cost of the bill for their lunch. The amount each friend paid was $4.25.

a. Write a division equation to represent c, the total cost, in dollars, of the bill for lunch.
b. Solve the equation to solve for the total cost of the bill.

Consider part *a* first.

First, rephrase the question to help you solve the problem: The total cost, c, divided by three equals 4.25, the amount each person paid.

Then, express this as an equation.

$$\frac{c}{3} = 4.25$$

Now consider part *b*.

Solve the equation by using the multiplication property of equality. Multiply both sides of the equation by 3.

$$\begin{aligned} \tfrac{c}{3} &= 4.25 \\ 3 \times \tfrac{c}{3} &= 3 \times 4.25 \end{aligned}$$

Next, rearrange the multiplication of fractions.

$$\frac{3}{3}c = 12.75$$

Now, simplify and solve.

$$\begin{aligned} 1c &= 12.75 \\ c &= 12.75 \end{aligned}$$

The answer is that the bill was $12.75.

Solve each equation.

Example 3

$\frac{x}{-2} = 5$

First, use the multiplication property of equality and multiply both sides of the equation by -2.

$$-2\frac{x}{-2} = -2 \times 5$$

Next, simplify and solve for x.

$$\begin{aligned} \tfrac{-2}{-2}x &= -10 \\ 1x &= -10 \\ x &= -10 \end{aligned}$$

The answer is $x = -10$.

Example 4

$\frac{y}{5} = 6$

First, use the multiplication property of equality and multiply both sides of the equation by 5.

$$5\frac{y}{5} = 5 \times 6$$

Next, simplify and solve for y.

$$\begin{aligned} \tfrac{5}{5}y &= 30 \\ 1y &= 30 \\ y &= 30 \end{aligned}$$

The answer is $y = 30$.

Example 5

$\frac{b}{-4} = -3$

First, use the multiplication property of equality and multiply both sides of the equation by -4.

$$-4\frac{b}{-4} = -4 \times -3$$

Next, simplify and solve for b.

$$\begin{aligned} \tfrac{-4}{-4}b &= 12 \\ 1b &= 12 \\ b &= 12 \end{aligned}$$

The answer is $b = 12$.

Review

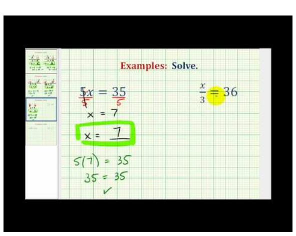
Solve each single variable division equation for the missing value.

1. $\frac{x}{5} = 2$
2. $\frac{y}{7} = 3$
3. $\frac{b}{9} = -4$
4. $\frac{b}{8} = -10$
5. $\frac{b}{8} = 20$
6. $\frac{x}{-3} = 10$
7. $\frac{y}{18} = -20$
8. $\frac{a}{-9} = -9$
9. $\frac{x}{11} = -12$
10. $\frac{x}{3} = -3$
11. $\frac{x}{5} = -8$
12. $\frac{x}{1.3} = 3$
13. $\frac{x}{2.4} = 4$
14. $\frac{x}{6} = 1.2$
15. $\frac{y}{1.5} = 3$

Review (Answers)

To see the Review answers, open this PDF file and look for section 7.10.

Resources



MEDIA

Click image to the left or use the URL below.

URL: http://www.ck12.org/flx/render/embeddedobject/182123

1.11 Two-Step Equations from Verbal Models

Learning Objectives

In this concept, you will learn to write two-step equations from verbal models.

Kara works as a babysitter for the neighborhood she lives in. She is saving money to plant an amazing sustainable home garden. For each babysitting job she took on, Kara charged \$4 for bus fare plus an additional \$8 for each hour she worked. On Saturday, Kara earned \$26 for the entire babysitting job.

Write an equation to represent this situation, where h is the total number of hours that Kara worked.

In this concept, you will learn to write two-step equations from verbal models.

Writing Two-Step Equations from Verbal Models

You solve equations like $3x = 5$, $\frac{x}{-2} = -4$, and $x + 31 = 8$ by doing one operation to both sides of the equation to isolate the variable.

But how can you solve the following equations?

$$\begin{aligned} 3x+5 &= 20 \\ \frac{x}{3}-2 &= 5 \end{aligned}$$

In these equations you need to do two operations to each side of the equation to isolate the variable. However, before you do this, look at some examples of word problems that involve two steps.

Here is an example.

Change the following word problem into an equation.

Six times a number, plus five is forty-one.

First, change the language into number and symbols. “Times” means multiplication, “a number” since it is not identified is your variable x, “plus” means addition, and the word “is” means equals.

$$6x+5=41$$

The answer is $6x+5=41$.

Here is another example.

Change the following word problem into an equation.

Four less than two times a number is equal to eight.

First, change the language into an equation. “Less than” means subtraction but be careful about the order. “Times” means multiplication, “a number” is your variable x, and “is equal to” means the same thing as equals.

$$2x-4=8$$

The answer is $2x-4=8$.

Examples

Example 1

Earlier, you were given a problem about Kara’s sustainable garden.

She is saving money to build one at home. She has a babysitting job where she earns \$8 and hour, but she also charges \$4 for bus fare. If she earned \$26 in total, can you write and equation to represent this.

First, let h be the number of hours Kara worked.

Next, re-phrase the text to make it easier to understand. Her total earnings were \$4 plus the number of hours she worked times 8. This equals \$26 the total amount earned.

Then, turn the language into numbers and symbols and write the equation. “Plus” means addition, “the number of hours” is the variable h, “times” means multiplication.

$$4+8h=26$$

The answer is $4+8h=26$.

Example 2

Write an equation for this statement.

A number divided by two, and then added to six is equal to fourteen.

First, change the language into numbers and mathematical symbols. “A number” is your variable x, “divided by” means division, “and then added to” means that after you divide you add, and “is” means equals.

Then, write the equation.

$$\frac{x}{2}+6=14$$

The answer is $\frac{x}{2}+6=14$.

Write an equation for each word problem.

Example 3

The product of five and a number, plus three is twenty-three.

First, translate the language into numbers and symbols. “The product of” means multiply what is in the parenthetical expression “five and a number,” “a number” is your variable x, “plus” means addition, and “is” means equals.

Then, write your equation.

$$5x+3=23$$

The answer is $5x+3=23$.

Example 4

Six times a number, minus four is thirty-two.

First, translate the language into numbers and symbols. “Times” means multiplication, “a number” is the variable x, “minus” means subtraction, and “is” means equals.

$$6x-4=32$$

The answer is $6x-4=32$.

Example 5

A number y, divided by 3, and then added to seven is ten.

First, translate the language into numbers and symbols. “A number y” is your variable y, “and then added to” means you divide and then add, and “is” means equals.

$$\frac{y}{3}+7=10$$

The answer is $\frac{y}{3}+7=10$.

Review

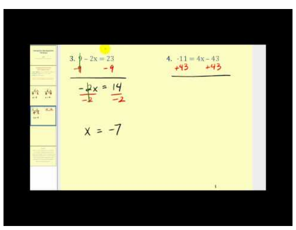
Write each statement as two-step equations.

1. Two times a number, plus seven is nineteen.
2. Three times a number, and five is twenty.
3. Six times a number, and ten is forty-six.
4. Seven less than two times a number is twenty-one.
5. Eight less than three times a number is sixteen.
6. A number divided by two, plus seven is ten.
7. A number divided by three, and six is eleven.
8. Two less than a number divided by four is ten.
9. Four times a number, and eight is twenty.
10. Five times a number, take away three is twelve.
11. Two times a number, and seven is twenty-nine.
12. Four times a number, and two is twenty-six.
13. Negative three times a number, take a way four is equal to negative ten.
14. Negative two times a number, and eight is equal to negative twelve.
15. Negative five times a number, minus eight is equal to seventeen.

Review (Answers)

To see the Review answers, open this PDF file and look for section 7.11.

Resources



MEDIA

Click image to the left or use the URL below.

URL: http://www.ck12.org/flx/render/embeddedobject/183592

1.12 Two-Step Equations and Properties of Equality

Learning Objectives

In this concept, you will learn to solve two-step equations.

As a mid-term project in a music class, some students have the opportunity to see a live jazz concert and describe the music to the class. The music teacher, Mr. Cooper, purchased some tickets. The service fee to buy tickets to the jazz concert is 9 dollars, and each ticket costs 12 dollars. If the total cost of the tickets was 93 dollars, can you figure out how many tickets Mr. Cooper his bought?

In this concept, you will learn to solve two-step equations.

Solving Two-Step Equations

Sometimes, when the numbers are small integers, you might be able to solve a two-step equation by thinking about it. For instance, can you solve $3x+3=9$?

Here the numbers are small. You can probably look at this equation and ask yourself, "What number times three plus three is nine?" The logical answer is 2. You can check your answer by substituting 2 in for x, to see if both sides of the equation are the same. If they are, then your work is accurate.

Let's plug in 2 for x to check that possible answer.

$$\begin{aligned} 3(2)+3 &= 9 \\ 6+3 &= 9 \\ 9 &= 9 \end{aligned}$$

Since it is true that $9 = 9$, the answer $x = 2$ works.

As equations get more complex it is important to use properties of equality to isolate the variable and solve the equation.

Here are the properties of equality you need to isolate terms and solve equations.

The Subtraction Property of Equality is used when you have an equation with addition in it. It states that you can subtract the same quantity from both sides of the equation without changing the equality.

The Addition Property of Equality is used when you have an equation with subtraction in it. It states that you can add the same quantity to both sides of the equation without changing the equality.

The Division Property of Equality is used when you have an equation with a variable multiplied by a number. It states that you can divide both sides of an equation by the same quantity (as long as that quantity is not equal to zero) without changing the equality.

The Multiplication Property of Equality is used when you have an equation with a variable divided by a number. It states that you can multiply both sides of an equation by the same quantity without changing the equality.

Let's look at an example and use properties of equality to isolate the variable and solve the equation.

$$2+3n = 11$$

There are two terms on the left side of the equation, 2 and $3n$.

The first step is to get the term with the variable, $3n$, by itself on one side of the equal $(=)$ sign.

In the equation, 2 is added to $3n$. So, you use the inverse of addition, which is subtraction, and subtract 2 from both sides of the equation.

You can do this because of the subtraction property of equality. That property states that in order to keep the values on both sides of the equation equal, whatever is subtracted from one side of the equation must also be subtracted from the other side.

Let's see what happens when we subtract 2 from both sides of the equation

$$\begin{aligned} 2+3n &= 11 \\ 2-2+3n &= 11-2 \\ 0+3n &= 9 \\ 3n &= 9 \end{aligned}$$

Now the problem is much more simple. You have reduced a two-step equation to a one-step equation.

Next, use the division property of equality and divide both sides of the equation by 3. Then simplify.

$$\begin{aligned} 3n &= 9 \\ \tfrac{3n}{3} &= \tfrac{9}{3} \\ 1n &= 3 \\ n &= 3 \end{aligned}$$

The answer is $n = 3$.

Here is another example.

Solve $2x - 5 = 11$ for x.

This is a two-step equation. The ultimate goal is to isolate the variable x. This will be easier to do if term with the variable, $2x$ is by itself on one side of the equation.

First, to get $2x$ by itself on one side of the equation, use the addition property of equality to add 5 to both sides of the equation.

$$\begin{aligned} 2x - 5 &= 11 \\ 2x - 5 + 5 &= 11 + 5 \\ 2x + 0 &= 16 \\ 2x &= 16 \end{aligned}$$

Now, the two-step equation is a one-step equation and is much easier to solve.

Next, use the division property of equality, and divide both sides of the equation by 2, to isolate the variable x.

$$\begin{aligned} \tfrac{2x}{2} &= \tfrac{16}{2} \\ 1x &= 8 \\ x &= 8 \end{aligned}$$

The answer is $x = 8$.

Here is another example.

Solve $\frac{x}{5} - 8 = 17$ for x.

First, use the addition property of equality to get $\frac{x}{5}$ by itself on one side of the equation. Add 8 to both sides of the equation.

$$\begin{aligned} \tfrac{x}{5} - 8 &= 17 \\ \tfrac{x}{5} + -8 + 8 &= 17 + 8 \\ \tfrac{x}{5} + 0 &= 25 \\ \tfrac{x}{5} &= 25 \end{aligned}$$

Now, the two-step equation has been reduced to a one-step equation. Since x is divided by 5, you need to use the inverse of division, multiplication, to isolate the variable x.

Next, use the multiplication property of equality, and multiply both sides of the equation by 5. Then simplify both sides of the equation.

$$\begin{aligned} \tfrac{x}{5} \times 5 &= 25 \times 5 \\ x \times \tfrac{5}{5} &= 125 \\ x \times 1 &= 125 \\ x &= 125 \end{aligned}$$

The answer is $x = 125$.

Examples

Example 1

Earlier, you were given a problem about Mr. Cooper and his music class.

He bought tickets to an amazing jazz concert for some students. There was a \$9 service fee per order, and each ticket cost \$12. The total cost for the order was \$93. Can you write an equation to represent this situation and then solve it?

First, let n be the number of tickets bought. The total cost is 9 plus 12 times the number of tickets bought. This total cost is 93. Translate this into an equation.

$$12n+9=93$$

The answer is $12n+9=93$.

Next, solve the two-step equation.

First isolate $12n$. Use the subtraction property of equality and subtract 9 from both sides of the equation.

$$\begin{aligned} 12n+9 &= 93 \\ 12n+9-9 &= 93-9 \\ 12n+0 &= 84 \\ 12n &= 84 \end{aligned}$$

Now, solve the one-step equation. Use the division property of equality and divide both sides of the equation by 12.

$$\begin{aligned} \frac{12n}{12} &= \frac{84}{12} \\ \frac{12}{12}n &= 7 \\ 1n &= 7 \\ n &= 7 \end{aligned}$$

The answer is Mr. Cooper bought 7 tickets.

Example 2

A landscaper charges \$35 for each landscaping job, plus \$20 for each hour worked. After one landscaping job, the landscaper charged \$95.

a. Write an algebraic equation to represent h, the number of hours that the landscaper worked on that \$95 job.
b. How many hours did that job take?

Consider part a first.

The landscaper earned \$20 for each hour worked on that job, so you multiply \$20 by h, the number of hours worked, to find how the landscaper charged for the hours worked, and then add the initial \$35. This equals the total charge of \$95.

The answer is $20h+35=95$.

Next, consider part b.

Solve the equation to find the number of hours the landscaper worked on that job.

First, to isolate the term $20h$ use the subtraction property of equality and subtract 35 from both sides of the equation.

$$\begin{aligned}
20h+35-35 &= 95-35 \\
20h+0 &= 60 \\
20h &= 60
\end{aligned}$$

The two-step equation has been reduced to a one-step equation.

Next, use the division property of equality to isolate the variable h, and divide both sides of the equation by 20.

$$\begin{aligned}
\frac{20h}{20} &= \frac{60}{20} \\
\frac{20}{20}h &= 3 \\
1h &= 3 \\
h &= 3
\end{aligned}$$

The answer is the landscaper worked for 3 hours on the \$95 job.

Solve each equation.

Example 3

$$5x+7=32$$

First, use the subtraction property of equality and subtract 7 from both sides of the equation.

$$\begin{aligned}
5x+7-7 &= 32-7 \\
5x+0 &= 25 \\
5x &= 25
\end{aligned}$$

Next, use the division property of equality and divide each side of the equation by 5.

$$\begin{aligned}
\frac{5x}{5} &= \frac{25}{5} \\
\frac{5}{5}x &= 5 \\
1x &= 5 \\
x &= 5
\end{aligned}$$

The answer is $x=5$.

Example 4

$$3a+9=39$$

First, use the subtraction property of equality and subtract 9 from both sides of the equation.

$$\begin{aligned} 3a+9-9 &= 39-9 \\ 3a+0 &= 30 \\ 3a &= 30 \end{aligned}$$

Next, use the division property of equality and divide both sides of the equation by 3.

$$\begin{aligned} \tfrac{3a}{3} &= \tfrac{30}{3} \\ \tfrac{3}{3}a &= 10 \\ 1a &= 10 \\ a &= 10 \end{aligned}$$

The answer is $a = 10$.

Example 5

$$\frac{y}{4} - 8 = 4$$

First, use the addition property of equality and add 8 to both sides of the equation.

$$\begin{aligned} \tfrac{y}{4} - 8 + 8 &= 4 + 8 \\ \tfrac{y}{4} + 0 &= 12 \\ \tfrac{y}{4} &= 12 \end{aligned}$$

Next, use the multiplication property of equality and multiply both sides of the equation by 4.

$$\begin{aligned} \tfrac{y}{4} \times 4 &= 12 \times 4 \\ y \times \tfrac{4}{4} &= 48 \\ y \times 1 &= 48 \\ y &= 48 \end{aligned}$$

The answer is $y = 48$.

Review

Solve each two-step equation for the unknown variable.

1. $3x+2=14$
2. $6y+5=29$
3. $7x+3=24$
4. $5x+7=42$
5. $6y+1=43$
6. $9a+7=88$
7. $11b+12=56$
8. $12x-3=21$
9. $4y-5=19$

10. $3a-9=21$
11. $5b-8=37$
12. $7x-10=39$
13. $6x-12=30$

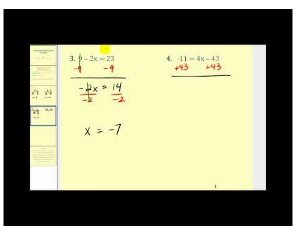
Write an equation for each word problem and then solve for the unknown variable.

14. Augusta sells t-shirts at the school store. On Tuesday, Augusta sold 7 less than twice the number of t-shirts she sold on Monday. She sold 3 t-shirts on Tuesday. Write an algebraic equation to represent m, the number of t-shirts August sold on Monday.
15. There are 19 green marbles in a box. The number of green marbles in the box is 6 more than half the number of red marbles in the box. Write an algebraic equation to represent r, the number of red marbles in the box.

Review (Answers)

To see the Review answers, open this PDF file and look for section 7.12.

Resources



MEDIA

Click image to the left or use the URL below.

URL: http://www.ck12.org/flx/render/embeddedobject/183592

1.13 Inequalities on a Number Line

Learning Objectives

In this concept, you will learn to graph inequalities on a number line.

A scientific center has been conducting research about bird populations that are being harmed by human development. Scientists are carefully gathering information on shrinking habitats and gathering data about the biology of the birds affected. A group of student scientists went to visit the center and learned about the beak length of various birds. They discovered that the beak length was always 7 inches or less. Can you represent the range of possible beak lengths on a number line?

In this concept, you will learn to graph inequalities on a number line.

Graphing Inequalities on a Number Line

An **inequality** is a mathematical statement that uses one of the following symbols: $>$, $<$, $\geq$, $\leq$, instead of an equals sign.

Here is what those symbols mean:

$>$ greater than

$<$ less than

$\geq$ greater than or equal to

$\leq$ less than or equal to

Let's look at an example to see how to interpret these symbols.

If you have the statement $x > 2$, what does it mean?

The inequality $x > 2$ has a variable, x, that represents an infinite set of numbers. Although x cannot be equal to 2, it can be any real number greater than 2. It is impossible to write out every single number that x can be. For this reason, you will see that a number line is a useful tool to represent the solution. For now, let's write out some of the numbers that are in the solution of $x > 2$.

To make this statement true, x can be any of the following numbers.

$$\left\{2.01, \frac{19}{9}, 2.988, 3, 4, 4.0000001, \frac{20}{3}, 1000, 1001.11111\cdots\right\}$$

You can start to see the range of numbers that make the statement $x > 2$ true. Though it is helpful to see some of the numbers in the solution for $x > 2$, it is best to represent the solution on a number line.

Let's look at an example where you graph an inequality.

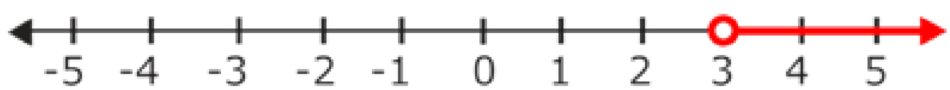
Graph the solution set for the inequality $x > 3$ on a number line.

To help complete this task, first draw a number line from -5 to 5, marking off ticks at integer intervals.

The inequality $x > 3$ is read as "x is greater than 3." So the solution of this inequality includes all numbers greater than 3. It does *not*, however, actually include 3. To show that the answer does not include 3, you use an open circle.

Next, draw an arrow showing all numbers greater than 3. The arrow should point towards the right because greater numbers are to the right on a number line.

The answer is



Here are some tips for graphing inequalities on a number line.

Use an open circle to show that a value is *not* a solution for the inequality. You will use open circles to graph inequalities that include the symbols $>$ or $<$.

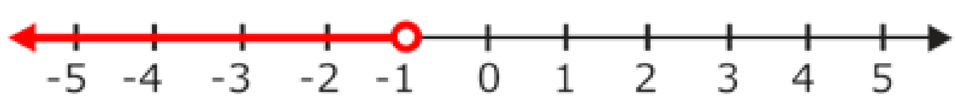
Use a closed circle to show that a value *is* a solution for the inequality. You will use closed circles to graph inequalities that include the symbols $\geq$ or $\leq$.

Here is another example.

Graph the solution for $x <$ -1 on a number line.

First, draw a number line from -5 to 5.

The inequality $x <$ -1 is read as "x is less than -1." The solutions of this inequality include all numbers less than -1, but not -1 itself, so draw an open circle at -1 to show that it is not a solution for this inequality. Then draw an arrow showing all numbers less than -1. The arrow should point towards the left because smaller numbers are to the left on a number line.



Examples

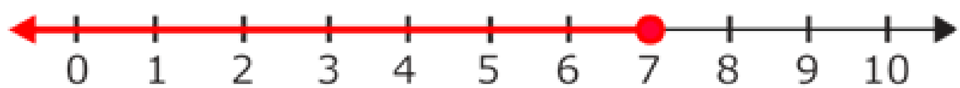
Example 1

Earlier, you were given a problem about the students going to the scientific center to study birds and their shrinking environment.

These students are collecting data on beak length and found that all the birds they looked at had beaks 7 inches or less. They need to represent the range of beak lengths on a number line.

Since there were beak lengths that were 7 inches you use a solid circle on 7 inches. They also found beaks less than 7 inches. So you draw an arrow to the left.

One possible solution is the following.



This is a fine answer, but can you improve it?

A beak length cannot be less than zero, so a better solution would be a closed circle at 7, with a line going to the left, but then stopping at an open circle at 0. This would show that beak lengths were also always greater than 0.

Example 2

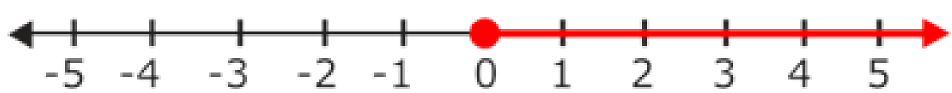
Graph the solution of $x \geq 0$ on a number line.

First, draw a number line from -5 to 5.

The inequality $x \geq 0$ is read as "x is greater than or equal to 0." So, the answer includes zero and all numbers that are greater than 0.

Draw a closed circle at 0 to show that 0 is in the solution for this inequality. Then draw an arrow pointing to the right, showing all numbers greater than 0 are also part of the solution set.

The answer is the graph below which shows the solution for the inequality $x \geq 0$.



Example 3

True or false: An open circle on a number line means that the circled number is not included in the solution set.

Open circles are a way to indicate that the circled number is not part of the solution.

The answer is true.

Example 4

True or false: An inequality does not include the number referenced in the solution set.

Inequalities include greater than or equal to $(\geq)$ and less than or equal to $(\leq)$. When these symbols are used the number referenced is included in the solution set.

The answer is false.

Example 5

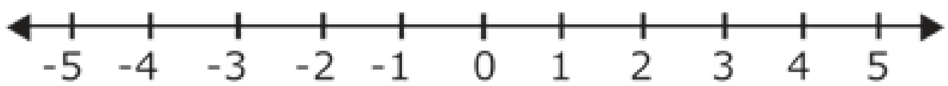
True or false: A closed circle on a graph means that the number is included in the solution set.

The answer is true.

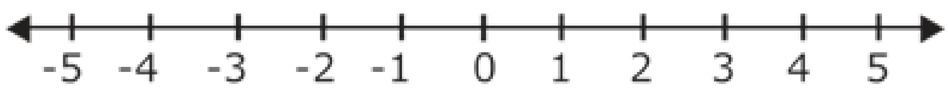
Review

Graph the solution for each inequality on the given number line.

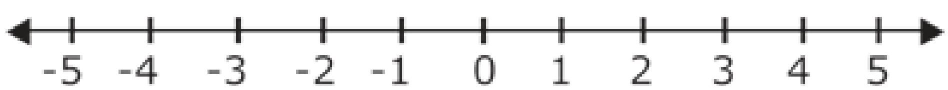
1. $x < -3$



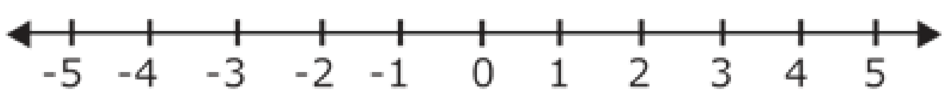
2. $x > -5$



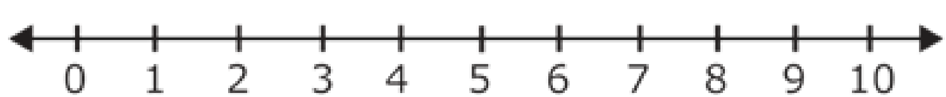
3. $n \leq 2$



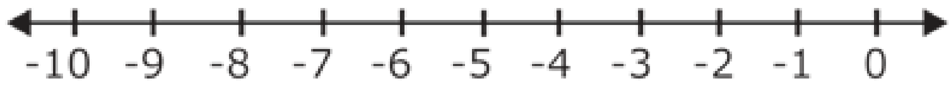
4. $1 \leq n$



5. $x > 6$

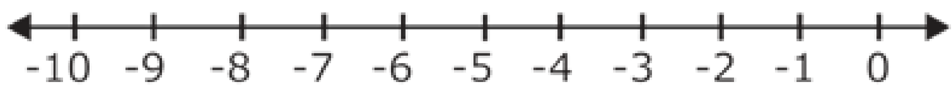


6. $n < -5$



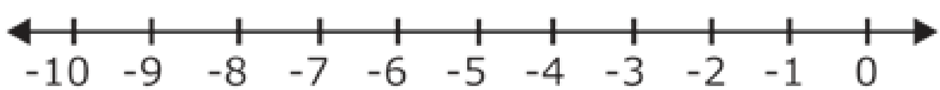
7. $n \leq -6$

8. $x \geq -9$

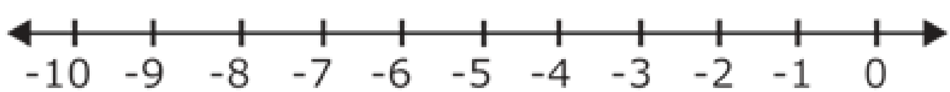


9. $x \leq -2$

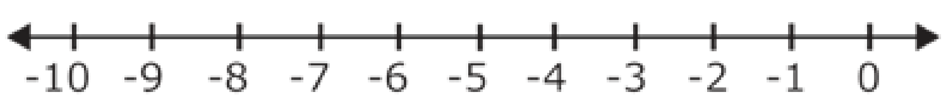
10. $x > -5$



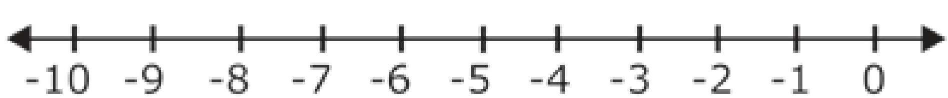
11. $x \leq -8$



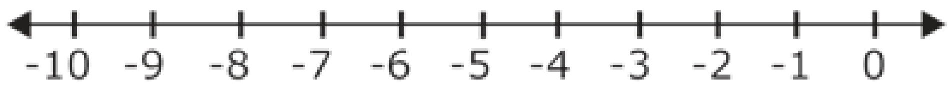
12. $x > -10$



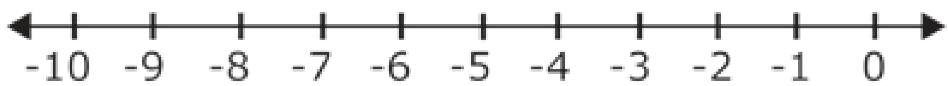
13. $x < -6$



14. $x > -7$



15. $x \leq -8$



Review (Answers)

To see the Review answers, open this PDF file and look for section 7.13.

1.14 Two-Step Inequalities

Learning Objectives

In this concept, you will learn to solve inequalities and graph solutions.

Marcia and Joe need to collect dragonflies and gather data about them for biology class. They need to capture at least 32 dragonflies for their project. They have already collected 3 dragonflies, and they can gather 2 dragonflies on average each hour. What is the minimum number of hours that Marcia and Joe need to search to get at least 32 dragonflies? Can you write an expression that models this information?

In this concept, you will learn to solve inequalities and graph solutions.

Solving Two-Step Inequalities

Inequalities are solved in almost the same way as regular equations. There are two small differences.

1. When solving for the variable in an inequality, if you multiply or divide each side of the equation by a negative number the direction of the inequality changes.
2. Unlike a regular equation, it is common to graph the solution set for an inequality on a number line.

Let's look at an example.

Solve this inequality and graph its solution on a number line $n-4 \leq 3$.

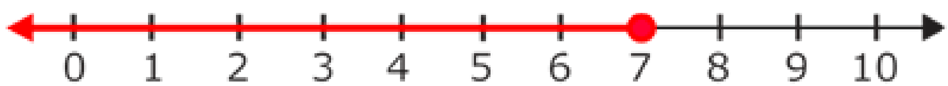
Solve the inequality as you would solve an equation, by using inverse operations and applying the properties of equality (e.g. the addition property of equality). Since the 4 is subtracted from n, add 4 to both sides of the inequality to solve it.

$$\begin{aligned} n-4 &\leq 3 \\ n-4+4 &\leq 3+4 \\ n+0 &\leq 7 \\ n &\leq 7 \end{aligned}$$

The answer is $n \leq 7$.

Next, graph the solution. The inequality $n \leq 7$ is read as "n is less than or equal to 7." So, the solution of this inequality includes 7 and all numbers that are less than 7.

Draw a number line from 0 to 10. Add a closed circle at 7 to show that 7 is a solution for this inequality. Then draw an arrow pointing to the left showing all numbers less than 7.



The solution set is graphed above.

Let's look at another example.

Solve this inequality and graph its solution on a number line $-2n < 14$.

Since n is multiplied by -2, divide both sides of the inequality by -2 to isolate the variable.

When working with inequalities if you divide or multiply both sides of the equation by a negative number the inequality must change directions. This means changing the inequality symbol from a "less than" symbol (<) to a "greater than" symbol (>).

$$\begin{aligned} -2n &< 14 \\ \tfrac{-2n}{-2} &> \tfrac{14}{-2} \\ 1n &> -7 \\ n &> -7 \end{aligned}$$

The answer is $n > -7$.

Next, graph the solution. The inequality $n > -7$ is read as "n is greater than -7." So, the solution of this inequality includes all numbers that are greater than -7, but not -7.

Draw a number line from -10 to 0. Add an open circle at -7 to show that -7 is not a solution for this inequality. Then draw an arrow pointing to the right which shows all numbers greater than -7 are included in the solution.

The answer is $n > -7$, and the graph of its solution is shown above.

Sometimes, it will take more than one step to solve an inequality. Solve these problems as if there were an equals sign, but remember that if you multiply or divide by a negative number the inequality will change direction.

Here is an example of a two-step inequality.

Solve the inequality $\frac{n}{3}+9 \geq -9$.

This problem is done just like a two-step equality.

First, isolate the $\frac{n}{3}$ term. Subtract 9 from both sides of the inequality.

$$\begin{aligned} \frac{n}{3}+9 &\geq -9 \\ \frac{n}{3}+9-9 &\geq -9-9 \\ \frac{n}{3} &\geq -18 \end{aligned}$$

Now, you have a one-step inequality, which is easier to solve. Since you will not be multiplying or dividing by a negative number the inequality will not change direction. Multiply both sides of the inequality by 3, to isolate the variable n.

$$\begin{aligned} \frac{n}{3} \times 3 &\geq -18 \times 3 \\ \frac{3}{3} \times n &\geq -54 \\ 1n &\geq -54 \\ n &\geq -54 \end{aligned}$$

The answer is $n \geq -54$.

Examples

Example 1

Earlier, you were given a problem about the dragonfly project.

Marcia and Joe need to gather at least 32 dragonflies for their project. They have 3 already and can collect 2 each hour. Can you write an inequality that describes this situation?

First, let h be the number of hours spent collecting. They collect two per hour, or $2h$ dragonflies. They start with 3. So, if the students work h hours they will have $3+2h$ dragonflies. They need at least 32 dragonflies, so $3+2h$ needs be equal or greater than 32.

The following inequality describes this situation.

$$3+2h \geq 32$$

Now, solve the inequality.

First, isolate $2h$ by subtracting 3 from both sides of the equation.

$$\begin{aligned} 3-3+2h &\geq 32-3 \\ 0+2h &\geq 29 \\ 2h &\geq 29 \end{aligned}$$

Next, isolate the variable h, and divide both sides by 2.

$$\begin{aligned}\frac{2h}{2} &\geq \frac{29}{2}\\ \frac{2}{2}h &\geq 14.5\\ 1h &\geq 14.5\\ h &\geq 14.5\end{aligned}$$

Finally, interpret the results. The students have to work at least 14.5 hours to collect 32 dragonflies. If you assume they need the full hour to collect the two dragonflies for that last hour, they should work 15 hours.

The answer is the students need to collect dragonflies for 15 hours.

Example 2

Antonio is buying milk for a breakfast event. Each container of milk costs \$3. At most, he can spend \$12 on milk for the event.

a. Write an inequality to represent, c, the number of containers of milk he can buy.
b. Could Antonio buy 4 containers of milk for the event? Explain.

Consider part a first.

Since each container of milk costs \$3, you can find the total cost, in dollars, of the milk he buys by multiplying the number of containers by 3. The phrase "at most" indicates that you should use the $\leq$ symbol.

The answer is $3c \leq 12$.

Note that the value of c must also be an integer greater than or equal to 0. Think about why that is for a moment.

The reason that the value of c must be an integer greater than or equal to zero is because Antonio cannot buy a negative number of containers nor can he buy a fraction of a container. Neither of those situations makes sense in real life. When using inequalities to represent real-life situations, you should always think about which values would make sense for the variable and which values would not make sense.

Next, consider part b.

Solve $3c \leq 12$ to help you answer if he can buy 4 containers of milk.

First, divide both sides of the equation by 3 to isolate the variable.

$$\begin{aligned}3c &\leq 12\\ \frac{3c}{3} &\leq \frac{12}{3}\\ \frac{3}{3}c &\leq 4\\ 1c &\leq 4\\ c &\leq 4\end{aligned}$$

According to the inequality above, the number of containers, c, that he can buy must be less than or equal to 4.

So, the answer is yes, Antonio can buy 4 containers of milk for the event.

Solve each inequality.

Example 3

$x - 4 < 10$

Add 4 to both sides of the equation to isolate the variable x.

$$\begin{aligned} x-4 &< 10 \\ x-4+4 &< 10+4 \\ x+0 &< 14 \\ x &< 14 \end{aligned}$$

The answer is $x < 14$.

Example 4

$2y+4 \geq 12$

First, subtract both sides of the equation by -4 to isolate the term $2y$.

$$\begin{aligned} 2y+4 &\geq 12 \\ 2y+4-4 &\geq 12-4 \\ 2y &\geq 8 \end{aligned}$$

Now you have a single-step inequality, which is easier to solve. Divide both sides of the equation by 2 to isolate the variable y.

$$\begin{aligned} \tfrac{2y}{2} &\geq \tfrac{8}{2} \\ \tfrac{2}{2}y &\geq 4 \\ 1y &\geq 4 \\ y &\geq 4 \end{aligned}$$

The answer is $y \geq 4$.

Example 5

$-4x \leq 16$

First, divide both sides of the equation by -4. You will have to change the direction of the inequality when you do this because you are dividing by a negative number.

$$\begin{aligned} -4x &\leq 16 \\ \tfrac{-4x}{-4} &\geq \tfrac{16}{-4} \\ \tfrac{-4}{-4}x &\geq -4 \\ 1x &\geq -4 \\ x &\geq -4 \end{aligned}$$

The answer is $x \geq -4$.

Review

Solve each inequality.

1. $x+4<10$
2. $x-3 \geq 7$
3. $b+5 \leq 15$
4. $a-7 \geq 14$
5. $4y > 20$
6. $6x \leq 18$
7. $-4y < -12$
8. $-5x < -20$
9. $\frac{x}{2} \leq 10$
10. $\frac{x}{5} \leq 6$
11. $2x+5 \geq 7$
12. $3y-2 \leq 4$
13. $3a-7 > 11$
14. $2b+9 < 39$
15. $2x-8 < 42$

Review (Answers)

To see the Review answers, open this PDF file and look for section 7.14.

1.15 Domain and Range of a Function

Learning Objectives

In this concept, you will learn to identify the domain and range of a simple linear function.

Joanna and Macy are helping with a research project about how fish waste is affecting the local environment. They know that fish length is related to the amount of waste produced by each fish. They also know that fish in the fish farm can be anywhere from 2 inches to 30 inches. The amount of waste (in mL) per day is approximately equal to .62 times the fish length.

Joanna and Macy are making a chart that captures what they know. Each fish length corresponds to a volume of waste. So far they recorded fish lengths of 2.8, 11.5, 10.7, 19.3, and 29. The corresponding amounts of waste are 1.736, 7.13, 6.634, 11.966, and 17.98.

Can you calculate the total daily amount of waste in the fish farm by creating a function and stating the domain range of that function?

In this concept, you will learn to identify the domain and range of a simple linear function.

Identifying the Domain and Range of a Function

A **function** is a rule that maps elements of one set, the **input**, to elements of another set, the **output**. In a function, an element of the input can be mapped to only one element in the output set. This means you cannot have one input being assigned to two different outputs.

One way to represent a function is as a set of ordered pairs. The first element in the ordered pair is an element of the input and it is assigned to the second element of the ordered pair, the output.

While the rule that does this assigning might not be apparent, you can still represent the function as a set of ordered pairs.

Let's look at an example of a function being described by a set of ordered pairs. Notice that braces, $\{\}$, are used to surround the set of ordered pairs.

$$\{(0,5),(1,6),(2,7),(3,8)\}$$

In $(1,6)$, 1 is an element in the input and it is being assigned to an element, 6, in the output. Each element, in the input $(0,1,2,3)$ is being assigned to only one element in the output $(5,6,7,8)$, so this list of ordered pairs does represent a function.

Let's take a look at another set of ordered pairs and determine if it represents a function or not and list the elements of the input and elements of the output.

$$\{(2,4),(5,3),(6,7),(2,8)\}$$

The elements of the input are $(2,5,6)$ and the elements of the output are $(4,3,7,8)$. However this set of ordered pairs does not represent a function. It violates one of the conditions of a function. The element 2, from the input is being assigned to two different outputs, 4 and 8. This doesn't work for functions.

More formally, the **domain** of a function is the set of all the elements in the input and the **range** is the set of all the elements in the output.

Let's look at another example.

Identify the domain and range of this function (or a rule that assigns an element of the input to an element of the output) represented as the following set of ordered pairs.

$$\{(0,-10),(2,-8),(4,-6),(6,-4)\}$$

The domain is the set of all the elements in the input, or all of the first elements of the ordered pairs.

The answer is the domain of this function is $\{0,2,4,6\}$.

The range is the set of all the elements in the output, or all of the second elements in the ordered pair.

The answer is the range of this function is $\{-10,-8,-6,-4\}$.

Examples

Example 1

Earlier, you were given a problem about Joanna and Macy, who are researching how waste from local fish farms is impacting the environment.

They have recorded fish lengths of 2.8, 11.5, 10.7, 19.3, and 29. The corresponding amounts of waste are 1.736, 7.13, 6.634, 11.966, and 17.98.

This information can be represented as a set or ordered pairs.

$$\{(2.8,1.736),(11.5,7.13),(10.7,6.634),(19.3,11.966),(29,17.98)\}$$

The five entries Joanna and Macy recorded represent a function with the input being the fish length and the output being amount of waste.

The domain of this function is the fish lengths, or $\{2.8, 11.5, 10.7, 19.3, 29\}$, and the range is the waste amount in mL, or $\{1.736, 7.13, 6.634, 11.966, 17.98\}$.

Example 2

Write the domain and range of this function, represented as a set of ordered pairs.

$$\{(1,3)(3,9)(4,6)(5,12)\}$$

The first value of each ordered pair is the domain of the function.

The second value of each ordered pair is the range of the function.

The answer is the domain is $\{1,3,4,5\}$ and the range is $\{3,9,6,12\}$.

Identify the domain and range of each function, represented as a set of ordered pairs.

Example 3

$$\{(1,3)(2,4)(5,7)(9,11)\}$$

The first value of each ordered pair is the domain of the function.

The second value of each ordered pair is the range of the function.

The domain is $\{1,2,5,9\}$ and the range is $\{3,4,7,11\}$.

Example 4

$$\{(8,12)(9,22)(4,7)(2,5)\}$$

The first value of each ordered pair is the domain of the function.

The second value of each ordered pair is the range of the function.

The domain is $\{8,9,4,2\}$, and the range is $\{12,22,7,5\}$.

Example 5

$$\{(8,9)(3,5)(7,6)(10,12)\}$$

The first value of each ordered pair is the domain of the function.

The second value of each ordered pair is the range of the function.

The domain is $\{8,3,7,10\}$, and the range is $\{9,5,6,12\}$.

Review

Identify whether the set of ordered pairs could represent a function or not.

1. $\{(1,3)(2,6)(2,5)(3,7)\}$
2. $\{(2,5)(3,6)(4,7)(5,8)\}$
3. $\{(6,1)(7,2)(8,3)\}$
4. $\{(5,2)(5,3)(5,4)(5,5)\}$
5. $\{(81,19)(75,18)(76,18)(77,19)\}$

For problems 6-10, state the domain of the function in numbers 1-5.

For problems 11-15, state the range of the function in numbers 1-5.

Review (Answers)

To see the Review answers, open this PDF file and look for section 7.15.

1.16 Input-Output Tables for Function Rules

Learning Objectives

In this concept, you will learn to evaluate a given function rule using an input-output table.

Soren is working on a project about the eating behavior of the Mandarin duck from East Asia. He has gathered some data about the Mandarin duck's weight and the approximate amount of plant matter it eats in a day. Soren put this information in a table and noticed a pattern in the duck's weight and the approximate amount of plant matter eaten.

TABLE 1.3:

Input (Mandarin duck weight in grams)	Output (Plant matter eaten in grams)
400	250
500	300
600	350
700	400

Can you write a rule that describes the data in this table?

In this concept, you will learn to evaluate a given function rule using an input-output table.

Using Input-Output Tables for Function Rules

An **input-output table**, like the one shown below, can be used to represent a function.

TABLE 1.4: 3 times x

Input number (x)	**Output number** (y)
0	0
1	3
2	6
3	9

Each pair of numbers in the table is related by the same function rule. That rule is: multiply each input number (x-value) by 3 to find each output number (y-value). You can use a rule like this to find other values for this function, too.

Now look at how to use a function rule to complete a table.

The rule for the input-output table below is: add 1.5 to each input number to find its corresponding output number. Use this rule to find the corresponding output numbers.

TABLE 1.5:

Input number (x)	**Output number** (y)
0	
1	
2.5	
5	
10	

To find each output number, add 1.5 to each input number. Then, write that output number in the table.

TABLE 1.6:

Input number (x)	**Output number** (y)	
0	1.5	$\leftarrow 0+1.5=1.5$
1	2.5	$\leftarrow 1+1.5=2.5$
2.5	4	$\leftarrow 2.5+1.5=4.0$
5	6.5	$\leftarrow 5+1.5=6.5$
10	11.5	$\leftarrow 10+1.5=11.5$

You can represent the information in this table as five ordered pairs from the function.

Now, write the answer in ordered pairs.

The answer is $(0,1.5)(1,2.5)(2.5,4)(5,6.5)(10,11.5)$.

Let's look at another example to show how to create a function table given a rule.

Let's say that the domain of a function is all real numbers. The rule for this function is: multiply each x-value (the input) by 4 and then subtract 2. Make a table that shows three inputs and three corresponding outputs for a function that follows this rule. Then, represent this information as ordered pairs.

First, choose three x-values for the table. You may choose any numbers in the domain of the function, but let's select some numbers that are easy to work with such as 1, 2, and 3.

Next, you take an input value (x-value), and apply the function rule to find the corresponding output value (y-value).

The rule for this function is: multiply each x-value (the input) by 4 and then subtract 2.

TABLE 1.7:

Input number (x)	Output number (y)	
1	2	$\leftarrow 4(1)-2=2$
2	6	$\leftarrow 4(2)-2=6$
3	10	$\leftarrow 4(3)-2=10$

You can represent the information in this table (the inputs and outputs of the function) as three ordered pairs.

The answer is $\{(1,2),(2,6),(3,10)\}$.

You can also write the rule for a function rule as an equation.

Here is an example.

The equation $y=\frac{x}{3}+1$ is a function. The input is the x-value and the corresponding y-value is the output. Use this rule to find the missing x and y values in the table below.

TABLE 1.8:

Input number (x)	Output number (y)
0	1
3	2
9	$\square$
$\square$	8

First, when x is 9, use the function to solve for y. You do this by substituting 9 in for x in $y=\frac{x}{3}+1$.

$$\begin{aligned} y &= \tfrac{x}{3}+1 \\ y &= \tfrac{9}{3}+1 \\ y &= 3+1 \\ y &= 4 \end{aligned}$$

One answer is when $x=9, y=4$. This means that $(9,4)$ is an ordered pair for this function.

Next, solve for the other missing value.

You know that $y=8$. So, plug that into the function and solve for x.

$$\begin{aligned} y &= \tfrac{x}{3}+1 \\ 8 &= \tfrac{x}{3}+1 \end{aligned}$$

Then, use the subtraction property of equality and subtract 1 from both sides of the equation.

$$\begin{aligned} 8-1 &= \tfrac{x}{3}+1-1 \\ 7 &= \tfrac{x}{3} \end{aligned}$$

Next, use the multiplication property of equality and multiply both sides of the equation by 3.

$$\begin{aligned}7 \times 3 &= \tfrac{x}{3} \times 3\\ 21 &= \tfrac{3}{3}x\\ 21 &= 1x\\ 21 &= x\end{aligned}$$

The answer is when the output value (y) is 8 the input value (x) is 21. This means that $(21,8)$ is an ordered pair for this function.

Examples

Example 1

Earlier, you were given a problem about Soren and the information he's gathering about Mandarin ducks.

Soren notices a pattern in the information he's collected so far. Can you write a rule that takes the input and gives the output number?

TABLE 1.9:

Input (Mandarin duck weight in grams)	Output (Plant matter eaten in grams)
400	250
500	300
600	350
700	400

Let x be the input, and y be the output.

You can notice that the outputs are about half the inputs, and as you look closer you may notice that the outputs are half the input plus 50. That is, the output, y, is $\frac{x}{2}+50$.

You can write this as a function taking an input value (x) to an output value (y).

$$y = \frac{x}{2} + 50$$

The answer for a rule or a function which takes duck weight as an input and gives plant matter eaten as an output is $y = \frac{x}{2} + 50$.

Example 2

Data about the number of chipmunks and the corresponding number of nuts eaten is presented in the following table.

TABLE 1.10:

Input (number of chipmunks)	Ouptut (nuts eaten)
12	6
10	5
8	4
6	3
4	2

Can you write a rule for a function that describes this information?

First, examine the inputs and outputs to see if there is a clear pattern. You can see that if you divide the input by 2 you get the output.

Next, let x be the input, and y be the output. You can then write a rule that describes this function.

The answer is $y = \frac{x}{2}$.

Solve each equation.

Example 3

Complete the table given the following rule: Add 2 to the input to get the output.

TABLE 1.11:

Input (x)	**Output** (y)
3	
5	
6	

First, add 2 to each input to find the output.

The answers are 5, 7 and 8.

Example 4

Write an equation that describes the function in Example 1.

Use the rule: add 2 to the input to get the output. Substitute x for the input, and y for the output.

The answer is $y = x + 2$.

Example 5

Create a table for the function $y = \frac{x}{2} + 3$. Use three values for the inputs.

First, it is a good idea to pick numbers for the inputs that will be easy to calculate. Let's take the numbers 2, 4, and 6 since they are all even numbers divisible by 2.

Next, plug each input value into x, in the function $y = \frac{x}{2} + 3$ and solve for y.

TABLE 1.12:

Input (x)	**Output** (y)
2	4
4	5
6	6

Review

Use the given rule or equation to complete the table.

1. The rule for the input-output below table is: multiply each input number by 7 and then add 2. Use this rule to

find the corresponding output numbers.

TABLE 1.13:

Input number (x)	**Output number** (y)
0	
1	
2	
3	
4	

2. The rule for this function table is: subtract 6 from each x-value to find each y-value. Use this rule to find the missing numbers in the table. Fill in the table with those numbers.

TABLE 1.14:

Input number (x)	**Output number** (y)
0	
6	
	7
	16

3. The equation $y = \frac{x}{2} - 1$ is a function. Use this function to find the missing values in the table below.

TABLE 1.15:

Input number (x)	**Output number** (y)
2	0
4	1
8	
	6

Determine if the rule provided would give the information in the input-output table. If it works write "yes" if not, write "no".

4. $2x$

TABLE 1.16:

Input number (x)	**Output number** (y)
1	2
2	5
3	7

5. $3x - 1$

Table 1.17:

Input number (x)	**Output number** (y)
1	2
2	5
3	8
4	11

6. $2x+1$

Table 1.18:

Input number (x)	**Output number** (y)
1	3
2	4
3	6
5	10

7. $4x$

Table 1.19:

Input number (x)	**Output number** (y)
0	0
1	4
2	8
3	12

8. $6x-3$

Table 1.20:

Input number (x)	**Output number** (y)
1	3
2	9
3	15

9. $2x$

Table 1.21:

Input number (x)	**Output number** (y)
0	0
1	2
2	4
3	6

10. $3x-3$

TABLE 1.22:

Input number (x)	**Output number** (y)
1	0
2	3
4	9
5	12

Create a table for each rule. Use five input values.

11. $7x$
12. $3x+1$
13. $5x-3$
14. $4x+3$
15. $4x-5$

Review (Answers)

To see the Review answers, open this PDF file and look for section 7.16.

1.17 Graphs of Linear Equations

Learning Objectives

In this concept, you will learn to graph linear functions on the coordinate plane.

Dana is collecting information about caterpillars for science class. She's comparing the lengths and widths of several caterpillars. Dana puts the data she has so far into a table. Dana is convinced there is a pattern. Can organize this information as a set of ordered pairs, graph it on a coordinate plane and write an equation that could model this?

x (width in cm)	y (length in cm)
2	2
3	4
4	6
5	8
6	10

In this concept, you will learn to graph linear functions on the coordinate plane.

Graphing Linear Functions

A **linear function** is a specific type of function. You may notice that the word "line" is part of the word "linear". That fact can help you remember that when a **linear function** is graphed on a coordinate plane, its graph will be a straight line.

You can represent a function as a set of ordered pairs, through a table, and as an equation. You can also take the information in ordered pairs or in a table and represent a function as a graph.

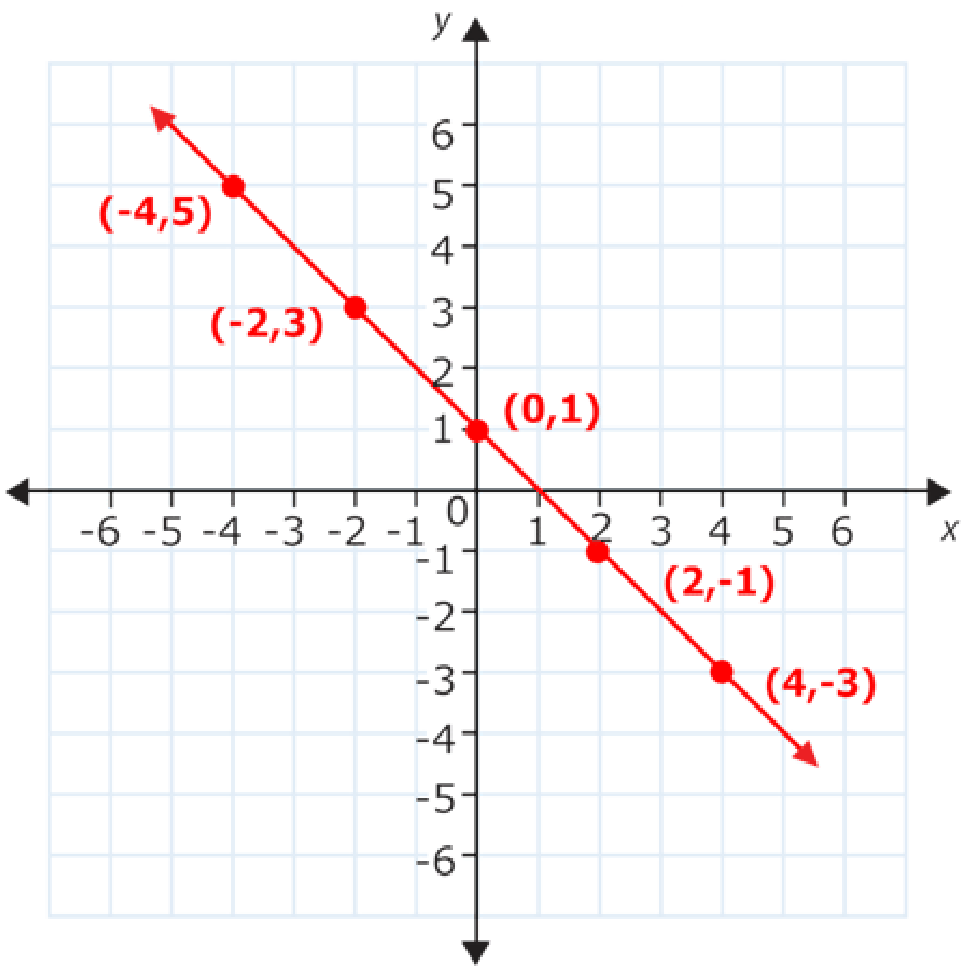
Let's look at an example.

The table of values below represents a function on a coordinate plane. On a coordinate plane, graph the linear function that is represented by the ordered pairs in the table below.

x	y
-4	5
-2	3
0	1
2	-1
4	-3

You can represent the information in this table as a set of ordered pairs $\{(-4,5),(-2,3),(0,1),(2,-1),(4,-3)\}$.

Plot those five points on the coordinate plane. Then, connect them as shown below.



Notice that the graph of this linear function is a straight line.

You can also graph a linear function if you are given an equation for that function. This will involve a few more steps. When you have an equation, you can use the equation to create a table. Then, plot several of the ordered pairs in the table and connect them with a line.

Here is another example.

The equation $y = 2x - 1$ is a linear function. Graph that function on a coordinate plane.

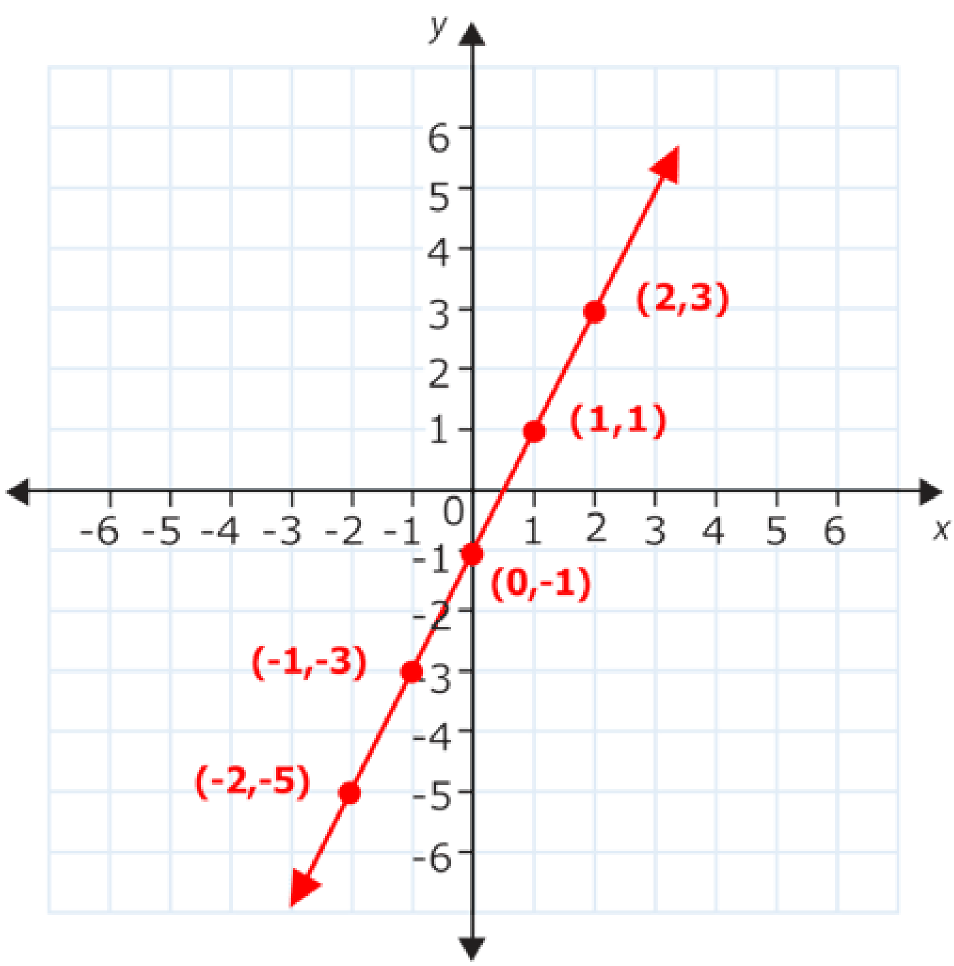
First, use the equation to create a table and find several ordered pairs for the function. It is a good idea to use some negative x-values, some positive x-values and 0. For example, you can create a table to find the values of y when x is equal to -2, -1, 0, 1, and 2.

TABLE 1.23:

x	y	
-2	-5	$2(-2) - 1 = -5$
-1	-3	$2(-1) - 1 = -3$
0	-1	$2(0) - 1 = -1$
1	1	$2(1) - 1 = 1$
2	3	$2(2) - 1 = 3$

The ordered pairs shown in the table are $(-2,-5), (-1,-3), (0,-1), (1,1)$ and $(2,3)$.

Plot those five points on the coordinate plane. Then connect them as shown below.



Examples

Example 1

Earlier, you were given a problem about Dana's project, which was comparing the lengths and widths of caterpillars.

She's put the data collected so far in a table (shown below). Can you plot these points and write the equation that models this information?

x (width in cm)	y (length in cm)
2	2
3	4
4	6
5	8
6	10

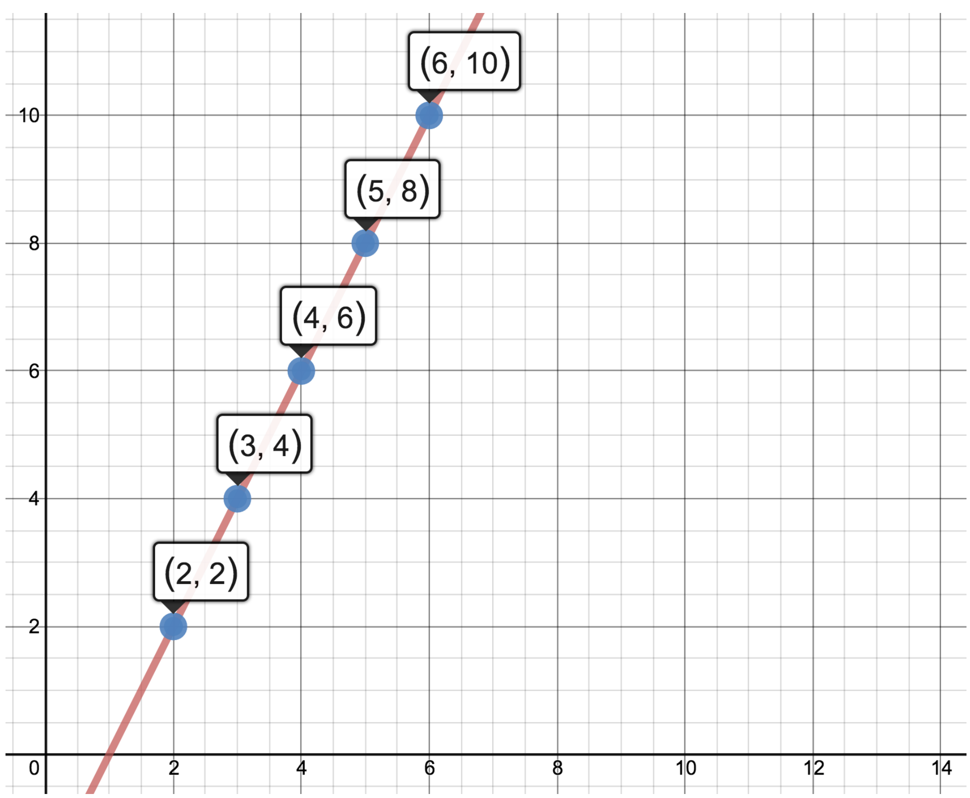
First, represent this information as a set of ordered pairs so that you can plot the points $\{(2,2),(3,4),(4,6),(5,8),(6,10)\}$.

Now, can you see a pattern in the table and then write the rule that describes it?

Notice that as x increases by 1, y increases by 2. So, you know that $2x$ is involved in the equation. But y is not quite $2x$. It is $2x-2$.

So the equation that models this information is $2x-2$.

Next, plot the points on the coordinate plane and draw a line through them. The graph is shown below.



Example 2

The table below represents inputs and outputs of a linear function. Can you represent this information as ordered pairs, figure out the equation for this function, and then graph the function?

x	y
1	5
2	10
3	15
4	20

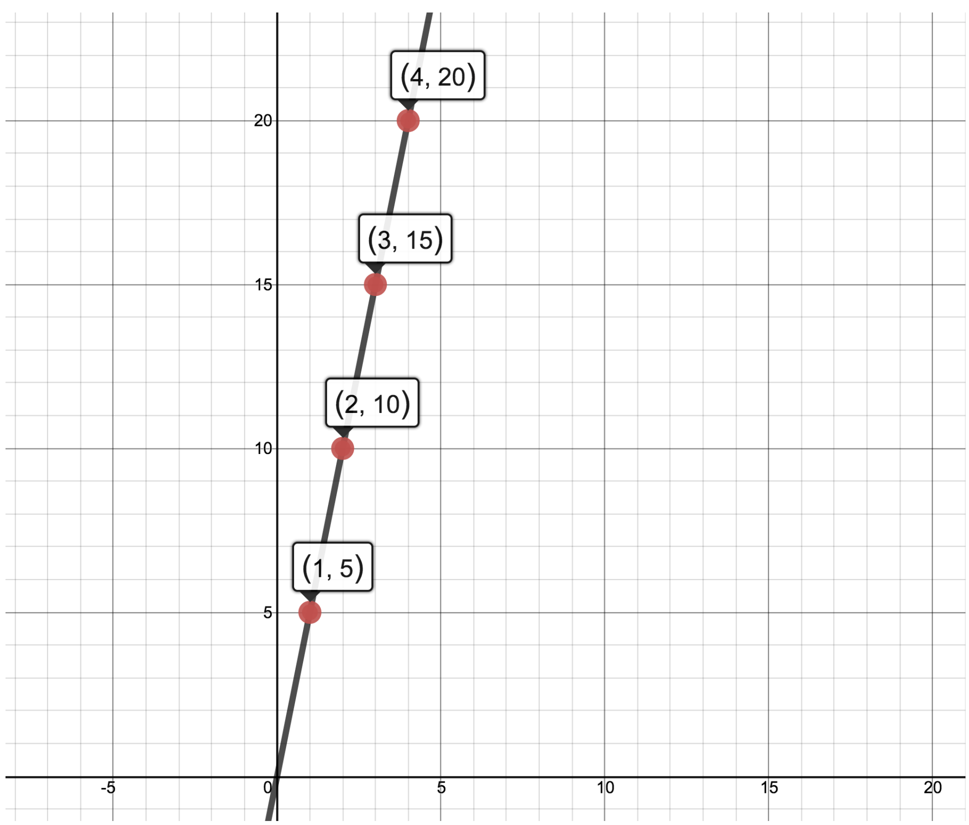
You can extract information from the table and represent the same information as a set of ordered pairs. The x-coordinate is the first value and the y-coordinate is the second value.

$$\{(1,5),(2,10),(3,15),(4,20)\}$$

Next, looking at the information in the table, you can see that when you multiply the x-value by 5 you get the y-value. The rule is multiply x by 5 to get y. You can write this as an equation.

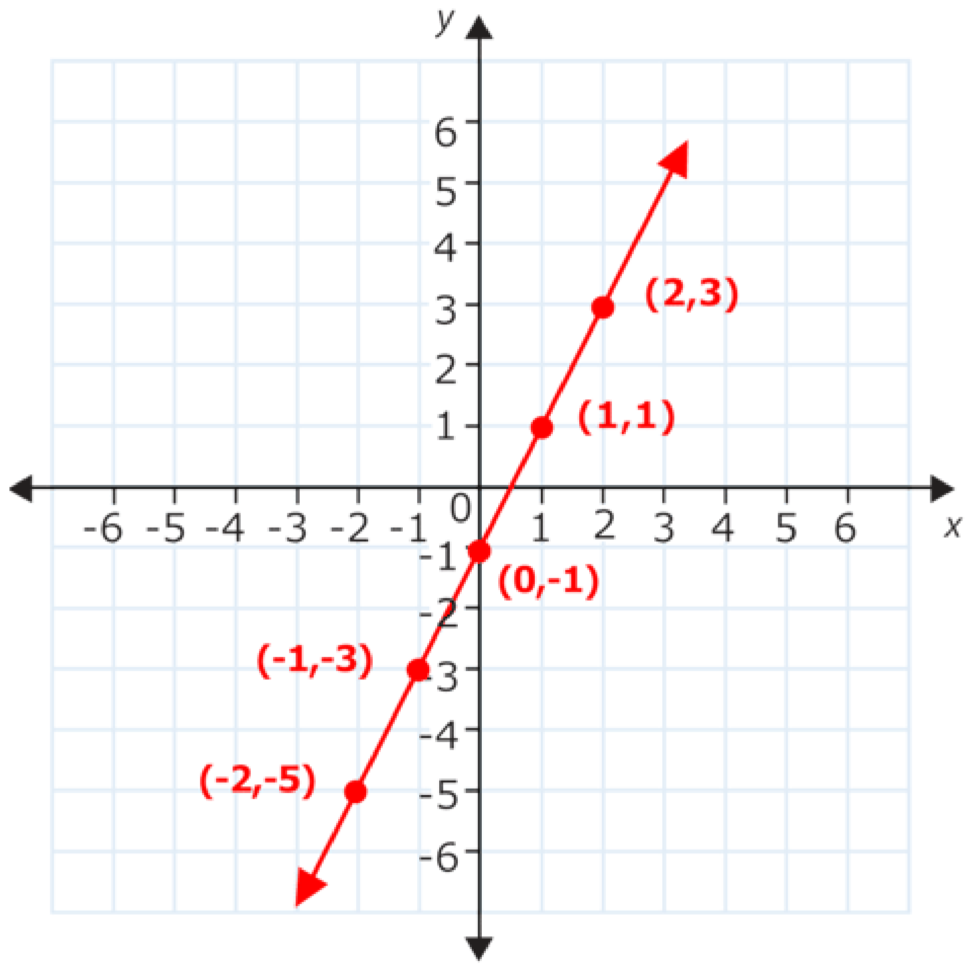
$$y = 5x$$

You can graph plot the coordinates $\{(1,5),(2,10),(3,15),(4,20)\}$ and draw a line through them to see the graph.



Answer the following questions about functions and coordinates.

Example 3



Is the function above increasing or decreasing?

Notice that as x increases y increases. Notice that every time you increase x by 1, y will always increase. In this case, y increases by two every time x increases by 1.

The answer is the function is increasing.

Example 4

In the point $(-3,4)$ is the x-value positive or negative?

The x-value is the first value in the coordinate. It is a negative number.

The answer is the x-value is negative.

Example 5

In $(-6,-7)$, which value is y-value?

The y-value is the second value in a coordinate, and it is equal to -7.

The answer is the y-value is -7.

Review

The information in the table represents points from a linear function. Plot the points in the table on a coordinate plane, and then draw a straight line through them to graph each function. Then identify the rule (equation) for the function.

1.

TABLE 1.24:

Input	Output
1	4
2	5
3	6
4	7

2.

TABLE 1.25:

Input	Output
2	4
3	6
4	8
5	10

3.

TABLE 1.26:

Input	Output
1	3
2	6
4	12
5	15

4.

TABLE 1.27:

Input	Output
9	7
7	5
5	3
3	1

5.

TABLE 1.28:

Input	Output
8	12
9	13
11	15
20	24

6.

TABLE 1.29:

Input	Output
3	21
4	28
6	42
8	56

7.

TABLE 1.30:

Input	Output
2	5
3	7
4	9
5	11

8.

TABLE 1.31:

Input	Output
4	7
5	9
6	11
8	15

9.

TABLE 1.32:

Input	Output
5	14
6	17
7	20
8	23

10.

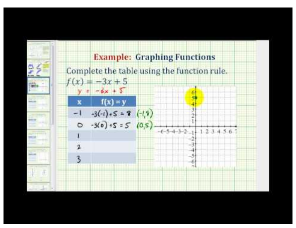
TABLE 1.33:

Input	Output
4	16
5	20
6	24
8	32

Review (Answers)

To see the Review answers, open this PDF file and look for section 7.17.

Resources



MEDIA
Click image to the left or use the URL below.
URL: http://www.ck12.org/flx/render/embeddedobject/183612

1.18 Linear and Nonlinear Function Distinction

Learning Objectives

In this concept, you will learn to distinguish between linear and nonlinear functions.

The championship bicycling race is a major event. Jess has trained for years to compete at this level and can go quite fast. Starting from rest, Jess can accelerate to 20 miles per hour. The coach records times and speeds and puts them in a table (shown below). The coach wants to measure Jess's progress and to see if there is a pattern in this data.

TABLE 1.34:

Input (time)	**Output (speed in mph)**
0	0
1	2
3	5
5	10
8	20

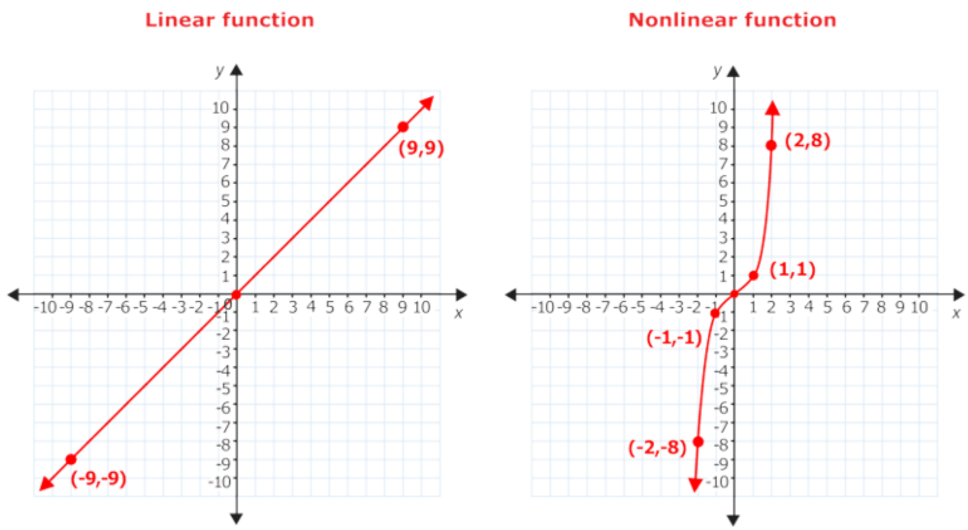
Can you plot this data on a coordinate plane, and then determine if this data could be modeled by a linear function?

In this concept, you will learn to distinguish between linear and nonlinear functions.

Distinguishing Between Linear and Nonlinear Functions

If you graph a linear function you will get a straight line. There are also **nonlinear functions**. If you graph the coordinates of a **nonlinear function** you will not get a straight line.

One of the easiest ways (but not the only way) to distinguish between a linear and a nonlinear function is to look at the graph of the function. Look at the two graphs below and you will see the difference between the two types of functions.



The first graph above is a linear function because its graph is a straight line. The second graph is a nonlinear function. Notice that the graph of this function is not a straight line. It is curved.

So, if you plot points from a function and cannot draw a straight line through them, then it is not a linear function.

Let's look at an example.

The equation $y = x^2$ is a function.

a. Plot several points to sketch a graph of the function.
b. Determine if the function linear or nonlinear.

First, consider part *a*.

Use the equation to create an input-output table. Use five x-values so you get a sense of what the function will look like when graphed. Pick positive and negative x-values centered around 0.

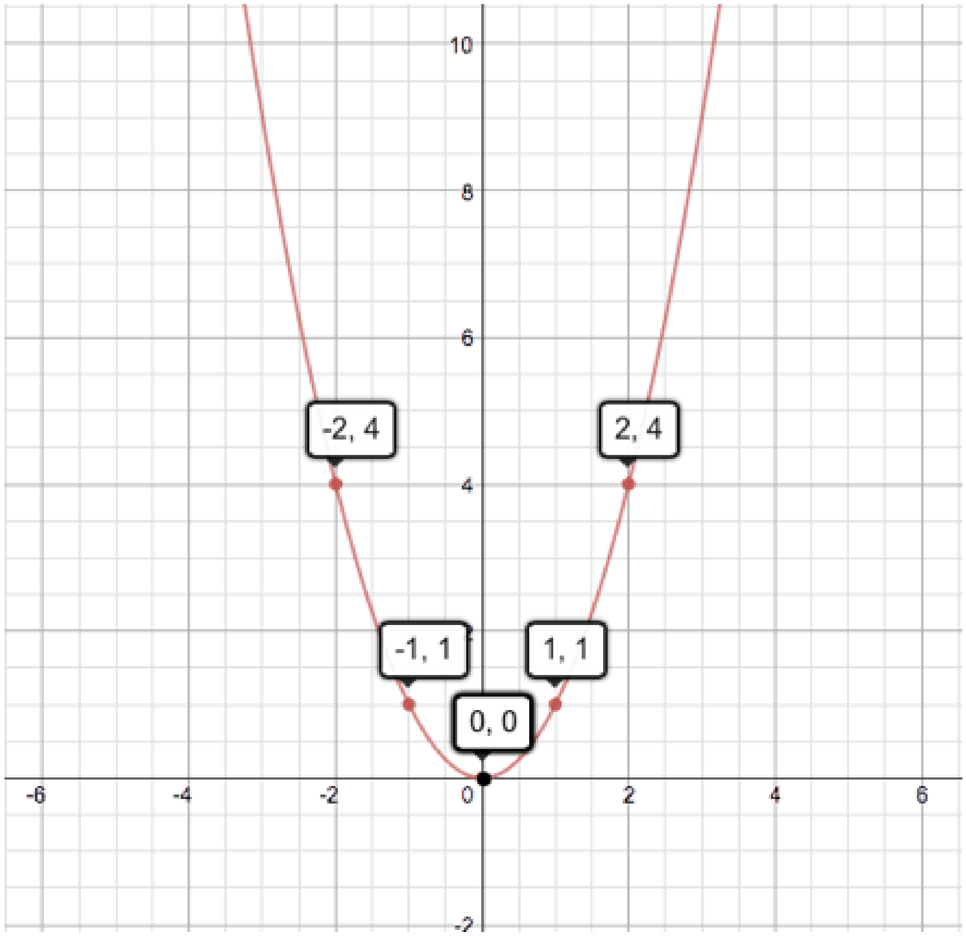
TABLE 1.35:

Input (x)	**Output** (y)	
-2	4	$y = (-2)^2 = (-2) \cdot (-2) = 4$
-1	1	$y = (-1)^2 = (-1) \cdot (-1) = 1$
0	0	$y = (0)^2 = (0) \cdot (0) = 0$
1	1	$y = (1)^2 = (1) \cdot (1) = 1$
2	4	$y = (2)^2 = (2) \cdot (2) = 4$

The ordered pairs shown in the table are $(-2,4), (-1,1), (0,0), (1,1)$ and $(2,4)$.

Plot those five points on the coordinate plane. Then, draw a curve to connect them.

The graph of $y = x^2$ is shown below. This kind of curve is called a parabola.



Next, consider part *b*.

Notice that you cannot connect these points with a straight line. You will need to draw a curved line to connect them.

The answer is $y = x^2$ is not a linear function.

Examples

Example 1

Earlier, you were given a problem about the bike race.

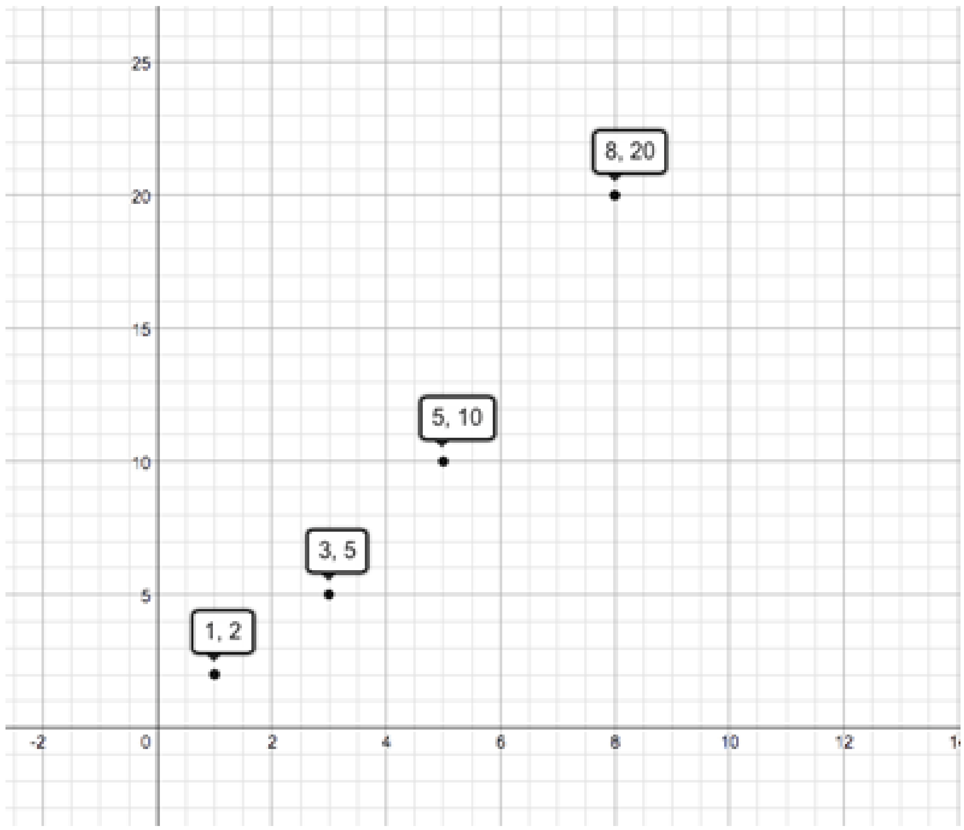
Jess's coach recorded times and speeds in the table below. You need to determine if this data could be modeled by a linear function.

TABLE 1.36:

Input (time)	Output (speed in mph)
0	0
1	2
3	5
5	10
8	20

First, write the information in this table as coordinates: $(0,0),(1,2),(3,5),(5,10),(8,20)$.

Then, plot these points (shown below).

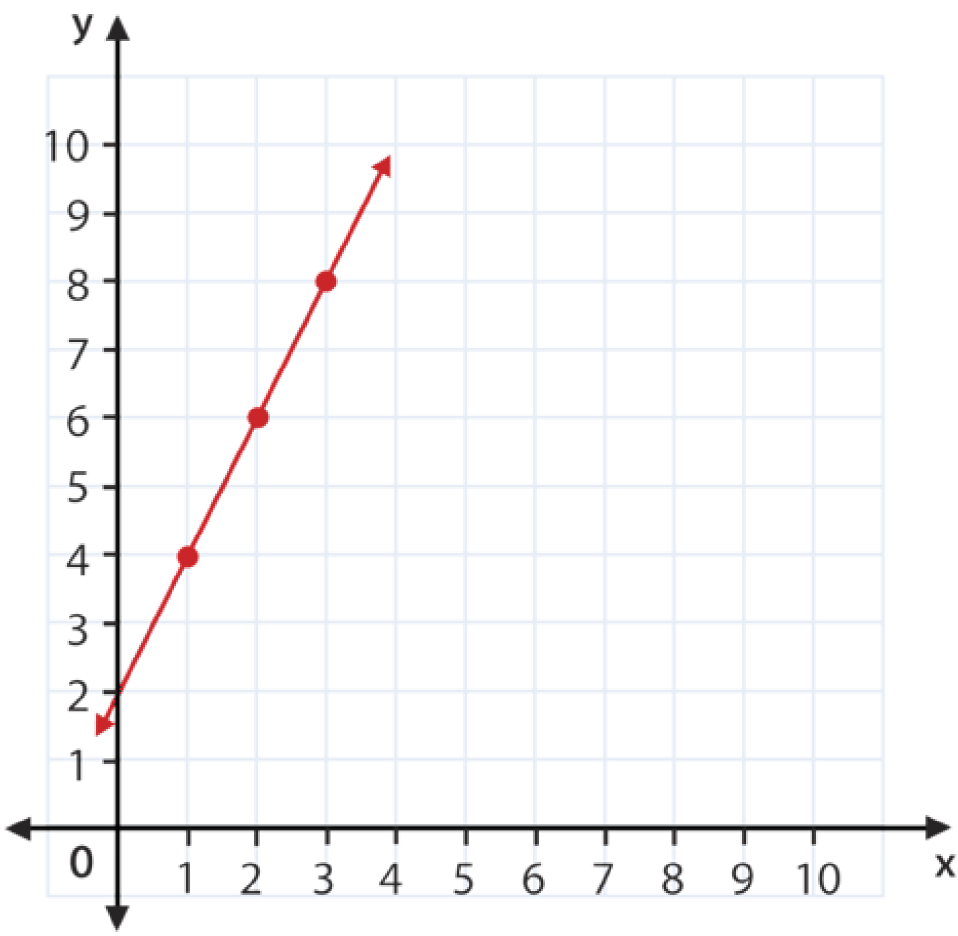


Next, determine if a linear function could model this data. Can you draw a straight (not curved) line that connects all of these points? You can get close, but a straight line will not go through all of the points, and is likely not the best model for this data.

The answer is no, this data is not part of a linear function.

Example 2

Look at the graph of a function below. Is this a linear or nonlinear function? Explain your answer. Then, write the coordinates of the ordered pairs highlighted on the graph.



The answer is that this is a linear function because the graph is a straight line.

The ordered pairs graphed are $(0,2),(1,4),(2,6),(3,8)$.

Several points from a function have been provided in the examples below. Determine if the points could be part of a linear function.

Example 3

$(0,2),(1,3),(2,4),(3,5)$

Plot these points on a coordinate plane. Since you can connect them with straight line (using a straight edge), they could be part of a linear function.

The answer is these points could be from a linear function.

Example 4

$(2,7),(3,5),(5,9),(9,6)$

Plot these points on a coordinate plane. Because you cannot connect these lines with a straight line, these points are not part of a linear function.

The answer is these points are from a nonlinear function.

Example 5

$(4,2),(6,4),(8,6),(10,8)$

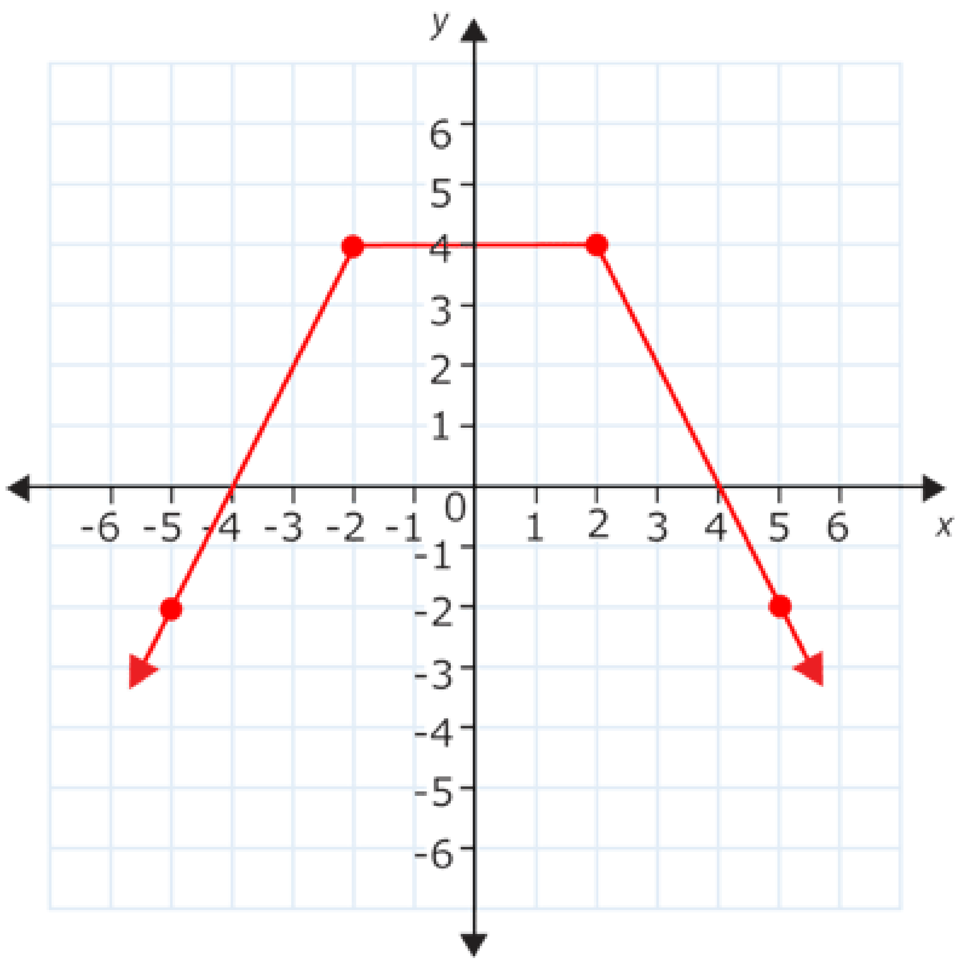
Plot these points on a coordinate plane. Since you can connect them with straight line (using a straight edge), they could be part of a linear function.

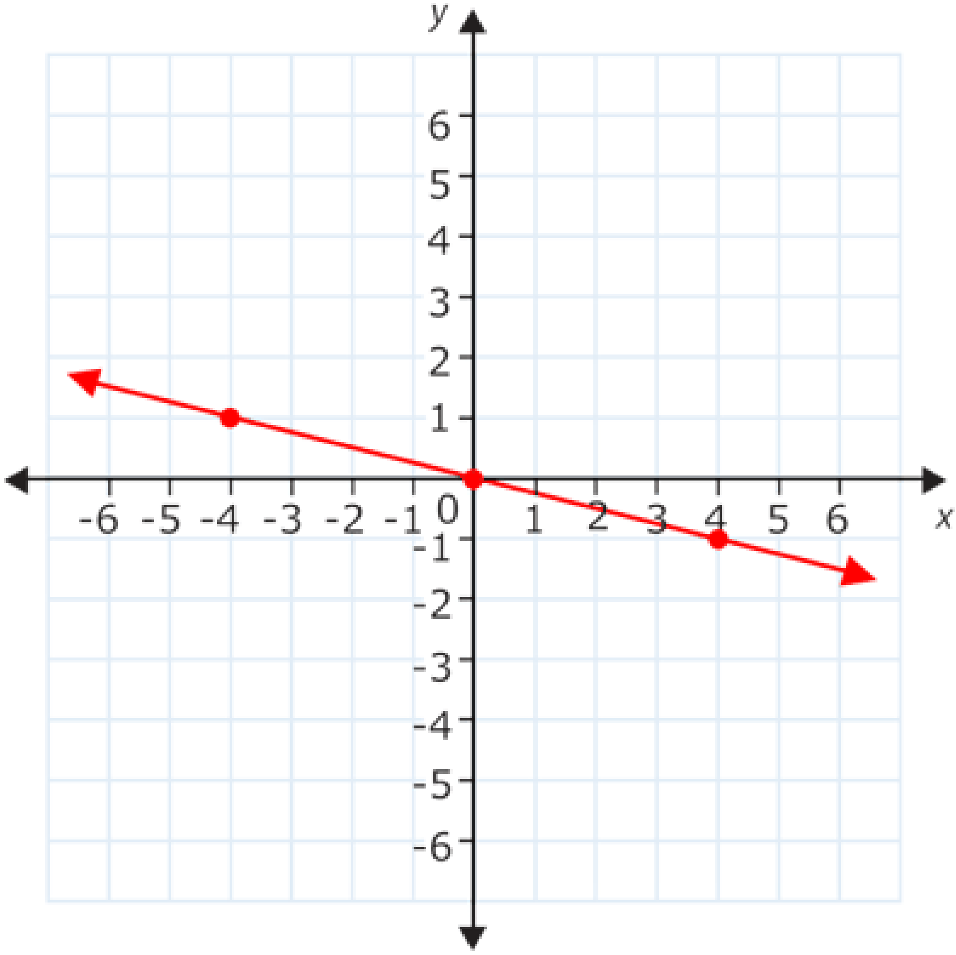
The answer is these points could be from a linear function.

Review

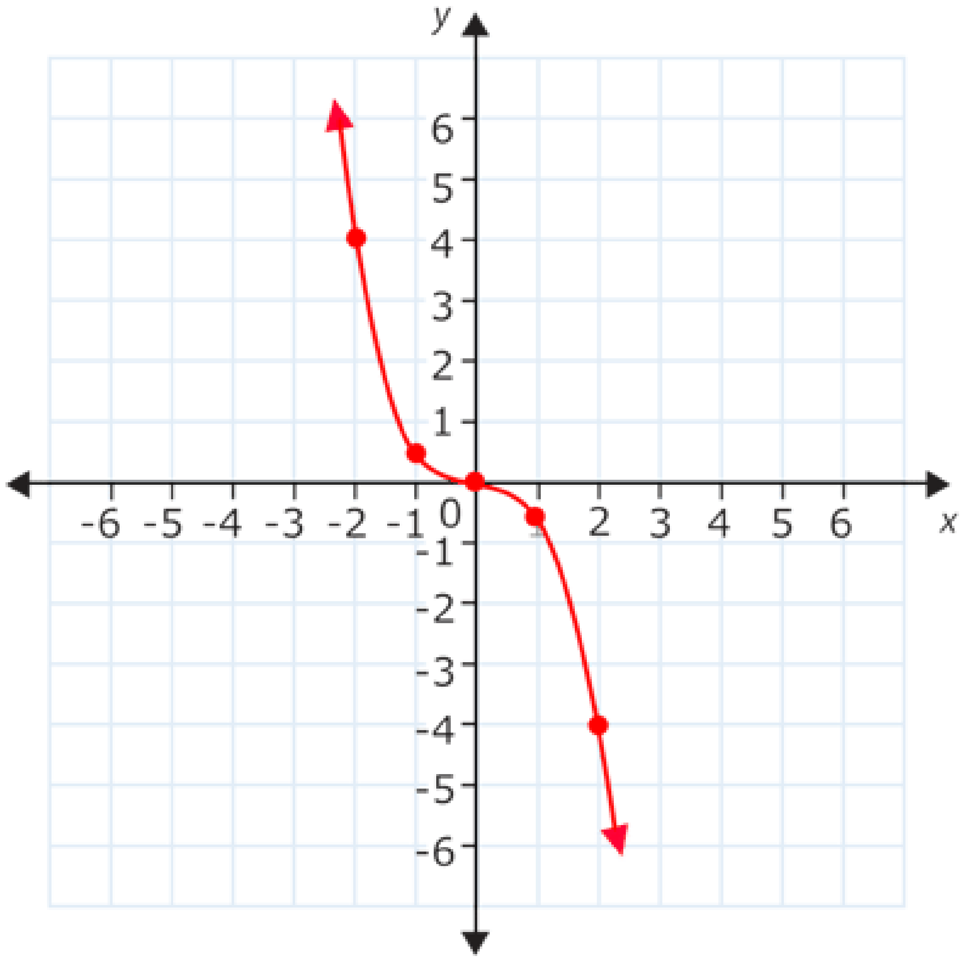
Determine if each graph shows a linear function or a nonlinear function.

1.





2.

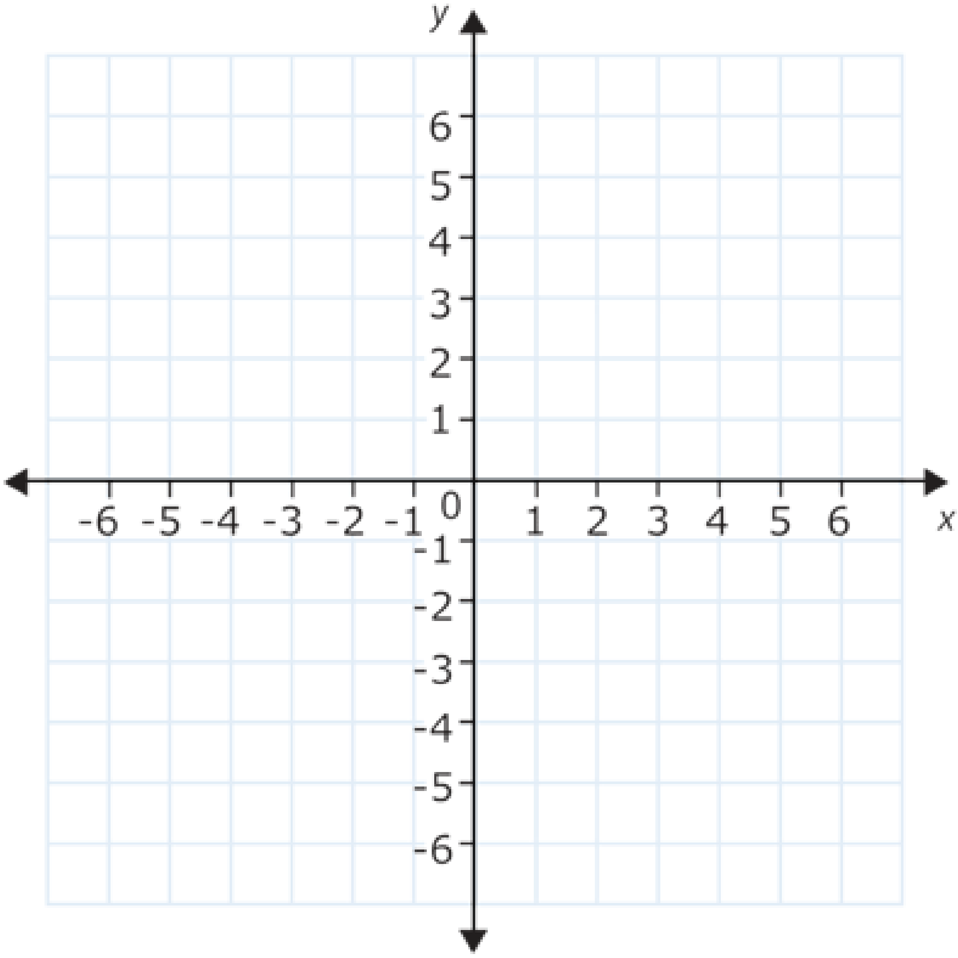


3.

4. The equation $y = \frac{x}{2} + 4$ is a function. Complete the table below to identify five ordered pairs for this function, and then plot the points on a coordinate plane. Then connect those points to sketch a graph for this function.

TABLE 1.37:

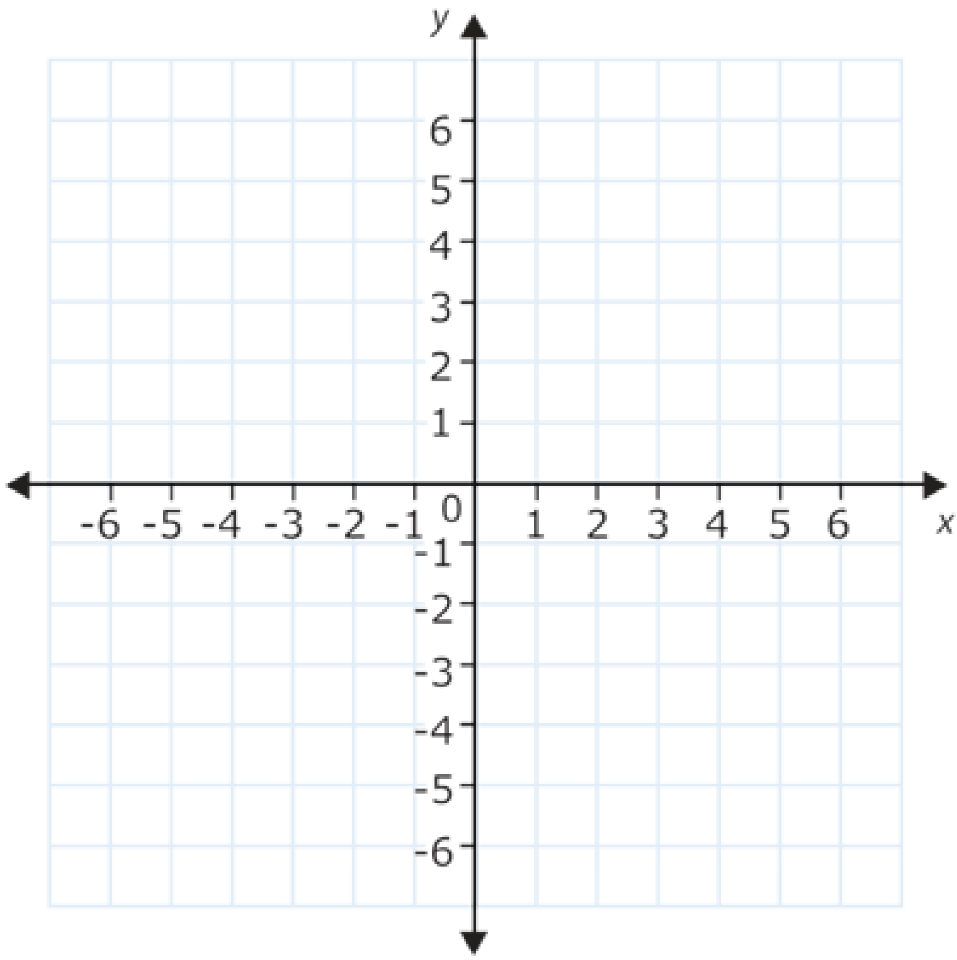
x	y
-4	
-2	
0	
2	
4	



5. Is the function you graphed in the previous question $\left(y = \frac{x}{2} + 4\right)$ a linear function or a nonlinear function?
6. The equation $y = x^2 + 2$ is a function. Complete the table below to identify five ordered pairs for this function. Plot those points on the coordinate plane below. Then connect those points to sketch a graph for this function.

TABLE 1.38:

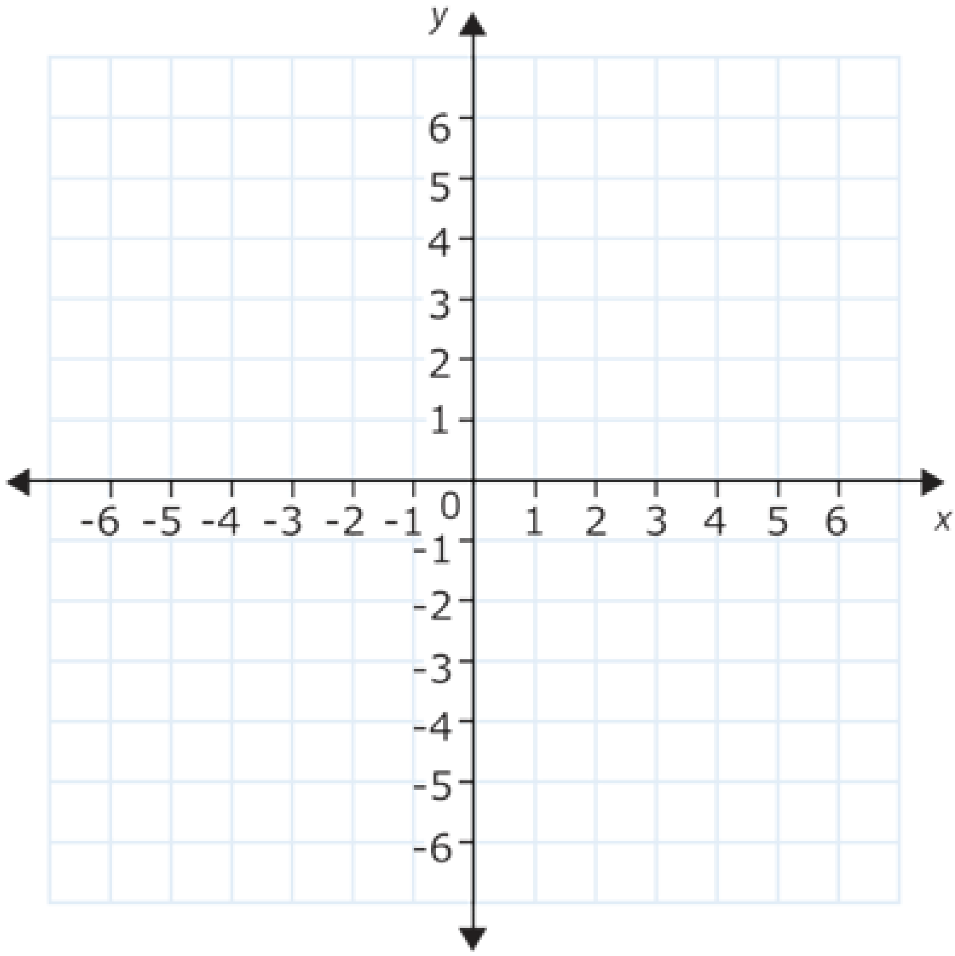
x	y
-2	
-1	
0	
1	
2	



7. Is the function you graphed in the previous question $(y = x^2 + 2)$ a linear function or a nonlinear function?

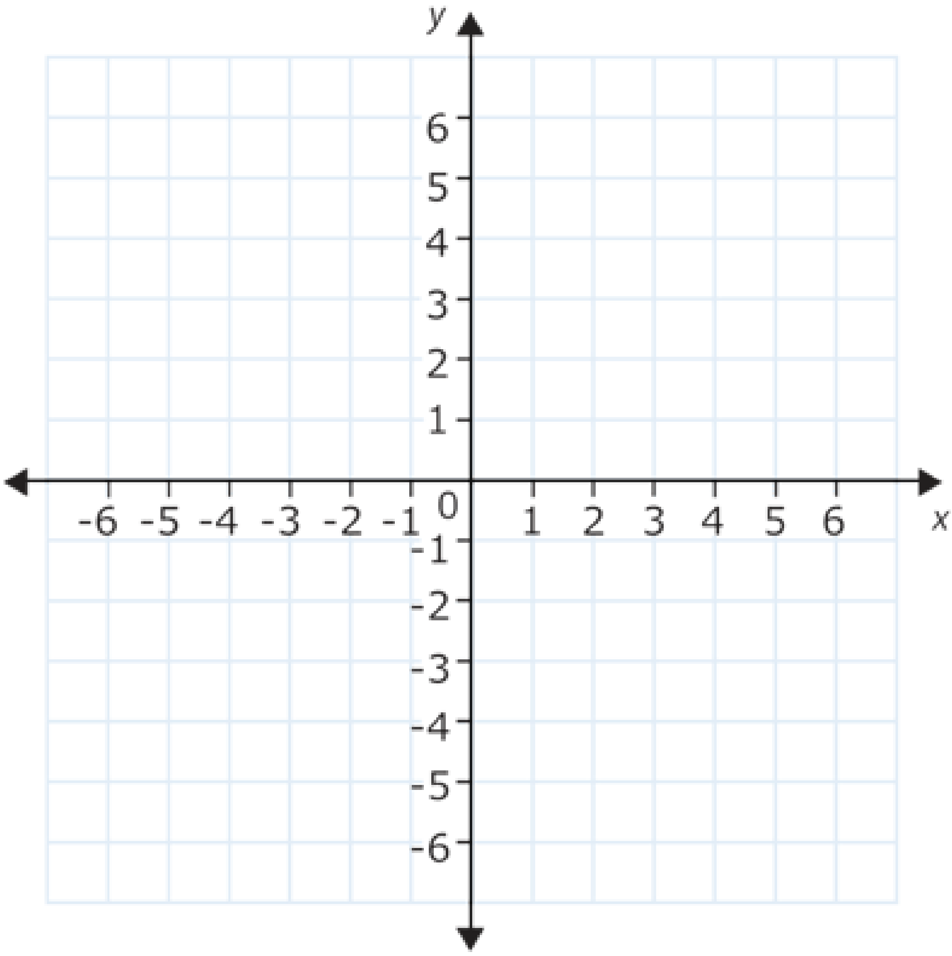
The rule for a linear function is: add 1 to each x-value to find each y-value.

8. Write an equation that represents this linear function.
9. Graph the function on this coordinate plane.



The rule for a linear function is: multiply each x-value by 2 and then subtract 2 to find each y-value.

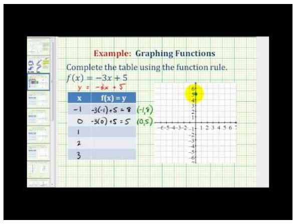
10. Write an equation that represents this linear function.
11. Graph the function on this coordinate plane.



Review (Answers)

To see the Review answers, open this PDF file and look for section 7.18.

Resources



MEDIA

Click image to the left or use the URL below.

URL: http://www.ck12.org/flx/render/embeddedobject/183612

1.19 Linear and Nonlinear Patterns of Change

Learning Objectives

In this concept, you will learn to model and solve real-world problems involving patterns of change and linear functions.

Some ocean birds dive from high above to catch fish swimming near the surface of the ocean. Samuel's favorite ocean bird is called a tern. After much observation, Samuel figures out that the equation $h = 19 - 16.1t^2$, where t is in seconds and h is in feet, approximately models the height of a tern on its descent into the ocean to catch a fish.

Is this function linear or nonlinear? Can you plot points using this equation and sketch a graph of the tern's descent into the ocean?

In this concept, you will learn to model and solve real-world problems involving patterns of change and linear functions.

Solving Problems Involving Patterns of Change

Functions can be used to model real-world data. Sometimes this data is linear, and can be modeled by a linear function, and sometimes the data is nonlinear, and thus cannot be modeled by a linear function.

Let's look at an example.

The table below shows how the total cost of buying tomatoes at the farmer's market changes depending on the number of pounds of tomatoes purchased.

TABLE 1.39:

Number of Ponds Purchased (x)	**Total Cost in Dollars** (y)

TABLE 1.39: (continued)

1	2
2	4
3	6
4	?
5	?

1. Assuming the pattern presented continues write an equation which models the relationship between the number of pounds purchased and the total cost.
2. Create a graph to represent the relationship between the number of pounds of tomatoes purchased, x, and the total cost, y.
3. Determine the cost of buying 5 pounds of tomatoes at the farmer's market.

Consider part *a* first.

You can see that as x increases by 1, y increases by 2. You may also notice that y is twice as much as x.

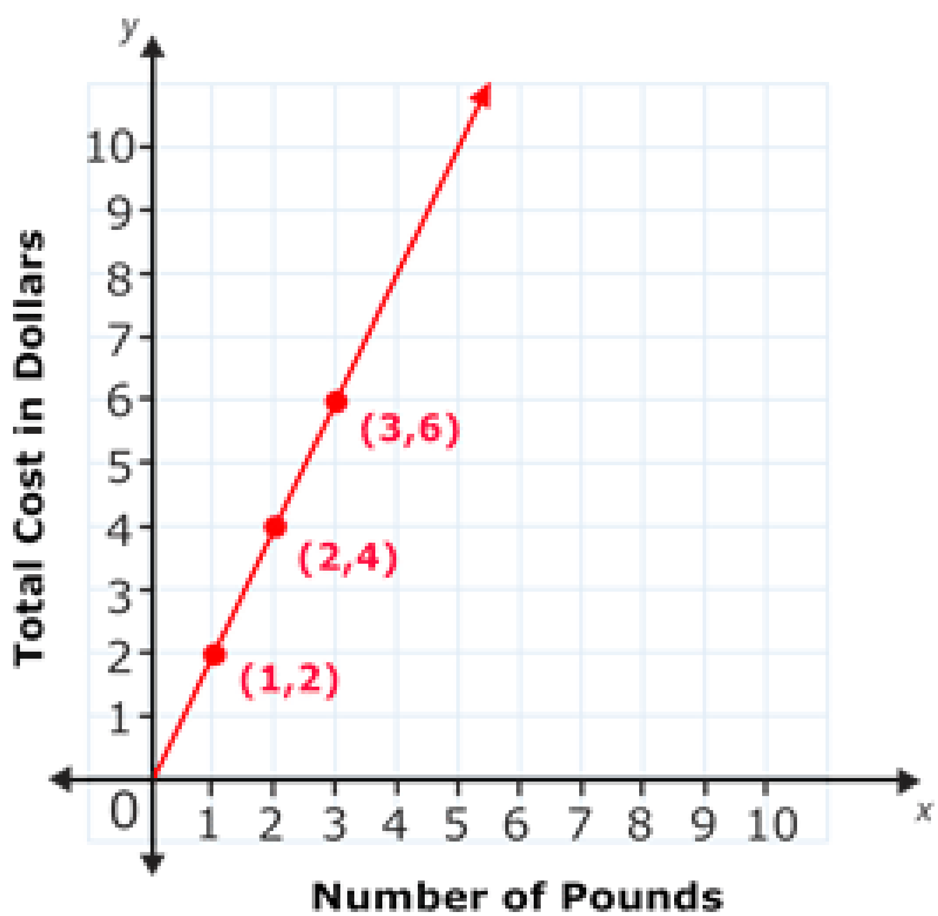
Since you assume that this pattern continues, the rule for this function is: multiply each x-value by 2 to find its corresponding y-value.

Now, translate those words to an equation. The equation is $y = 2x$.

Next, consider part *b*.

You need to make a graph that represents the function $y = 2x$.

First, plot the coordinates $(1,2),(2,4)$ and $(3,6)$ from the table. Then, draw a line through the points.



The graph above represents the relationship between the number of pounds of tomatoes purchased, x, and the total cost, y.

Finally, consider part c.

There are two ways to approach finding what y is, when $x = 5$.

One way, is to use the equation $y = 2x$ which models this data. Then, you just plug in 5 for x.

$$\begin{aligned} y &= 2x \\ y &= 2(5) \\ y &= 10 \end{aligned}$$

The answer is the total cost of 5 pounds of tomatoes is 10 dollars.

The other way to solve part c, is to complete the pattern (which you are assuming continues) in the table. The pattern is that if x increases by 1, y increases by 2.

The completed table is shown below.

TABLE 1.40:

Number of Pounds Purchased (x)	**Total Cost in Dollars** (y)
1	2
2	4
3	6
4	8
5	10

Looking at the table, you can see that the answer is the total cost of 5 pounds of tomatoes is 10 dollars.

Examples

Example 1

Earlier, you were given a problem about Samuel's tern.

The tern dives into the ocean to catch fish. On one such decent, the bird starts from 19 feet in the air and dives down to the surface of the ocean. Its descent can be modeled by the equation below.

$$h = 19 - 16.1t^2$$

In this equation, t is in seconds and h is in feet.

Can you make a table from this equation?

First, start when $t = 0$ and then go by small increments of time. Use increments of .2 and then round answers to the nearest tenth. This descent happens quickly.

TABLE 1.41:

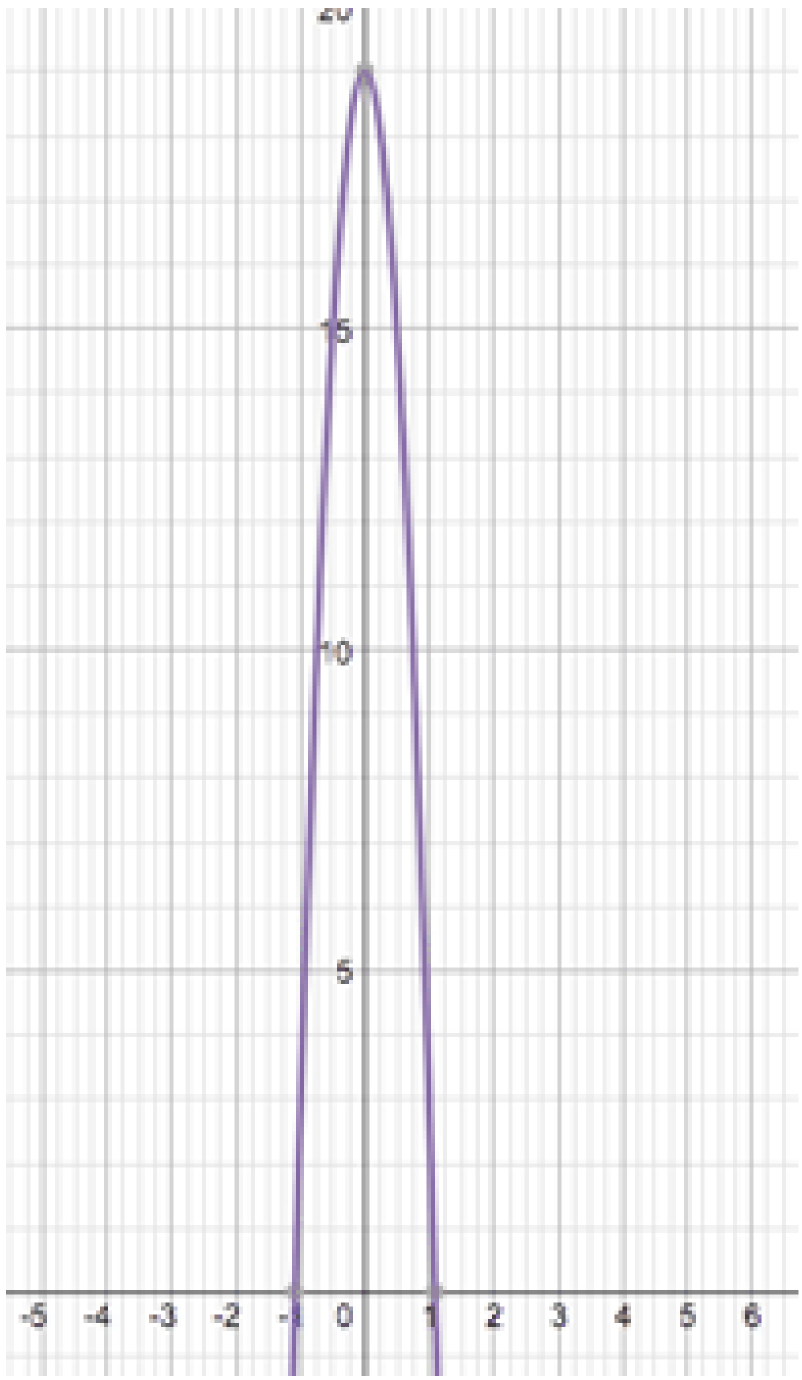
Height from ocean in feet (h)	**Time in seconds** (t)
0	19

TABLE 1.41: (continued)

.2	18.4
.4	16.4
.6	13.2
.8	8.7
1	2.9

Next, can you graph this data and connect the points?

The graph below represents this data.



For this situation, it makes sense to only look at values when $t > 0$ and $h > 0$.

Finally, determine if this graph is linear or nonlinear.

Since, a straight line does not go through the points from the table, the graph is nonlinear. You may also notice that for a constant change in x of .2, y does not have a constant rate of change.

Example 2

Is the following true or false?

If a function is linear, then, for each constant increase in x, there is a constant increase in y. That is, every time x increases by a constant number, y will increase by a constant number.

True. Plot points where, every time x increases by a constant number, y increases by a constant number. For example, try plotting points where every time x increases by 3, y increases by 5. You can connect these points with a straight line, so it is a linear function.

Solve for y using the given x, using the equation $y = 2x$.

Example 3

When $x = 4$

$$\begin{aligned} y &= 2x \\ y &= 2(4) \\ y &= 8 \end{aligned}$$

The answer is $y = 8$.

Example 4

When $x = 4.5$

$$\begin{aligned} y &= 2x \\ y &= 2(4.5) \\ y &= 9 \end{aligned}$$

The answer is $y = 9$.

Example 5

When $x = 5.5$

$$\begin{aligned} y &= 2x \\ y &= 2(5.5) \\ y &= 11 \end{aligned}$$

The answer is $y = 11$.

Review

Look at the information in each table and determine whether the data could be from a linear function or not.

1.

TABLE 1.42:

x	y
0	2
1	3
2	5
4	4

2.

TABLE 1.43:

x	y
1	3
2	5
3	7
4	9

3.

TABLE 1.44:

x	y
2	6
3	9
5	15
6	18

4.

TABLE 1.45:

x	y
2	3
3	4
6	7
8	9

5.

TABLE 1.46:

x	y

TABLE 1.46: (continued)

8	4
6	12
2	8
0	0

6.

TABLE 1.47:

x	y
0	3
1	4
2	5
6	9

7.

TABLE 1.48:

x	y
5	11
4	9
3	7
2	5

8.

TABLE 1.49:

x	y
1	7
3	4
2	9
5	8

9.

TABLE 1.50:

x	y
1	3
2	6
4	12
6	18

10.

TABLE 1.51:

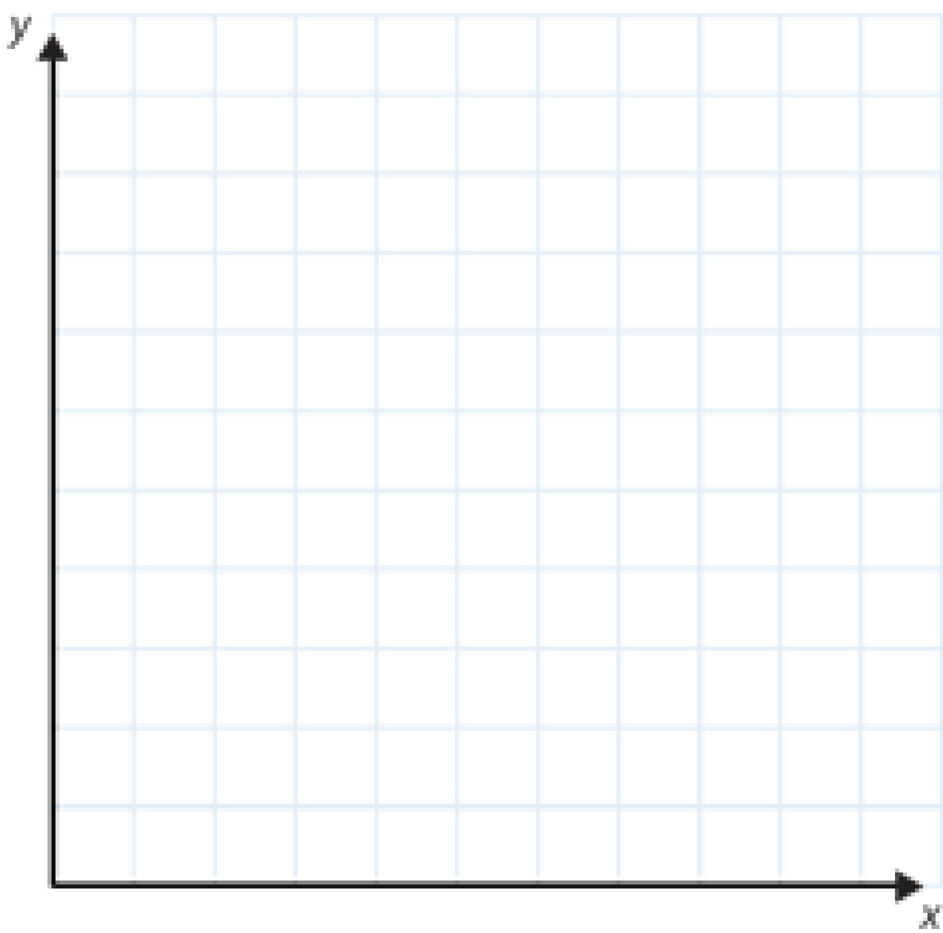
x	y
4	2
5	3
6	5
7	1

The table below shows how the total cost of buying gasoline at Gary's Gas Station changes depending on the number of gallons purchased.

TABLE 1.52:

Number of Gallons Purchased (x)	**Total Cost in Dollars** (y)
0	0
1	3
2	6
3	?

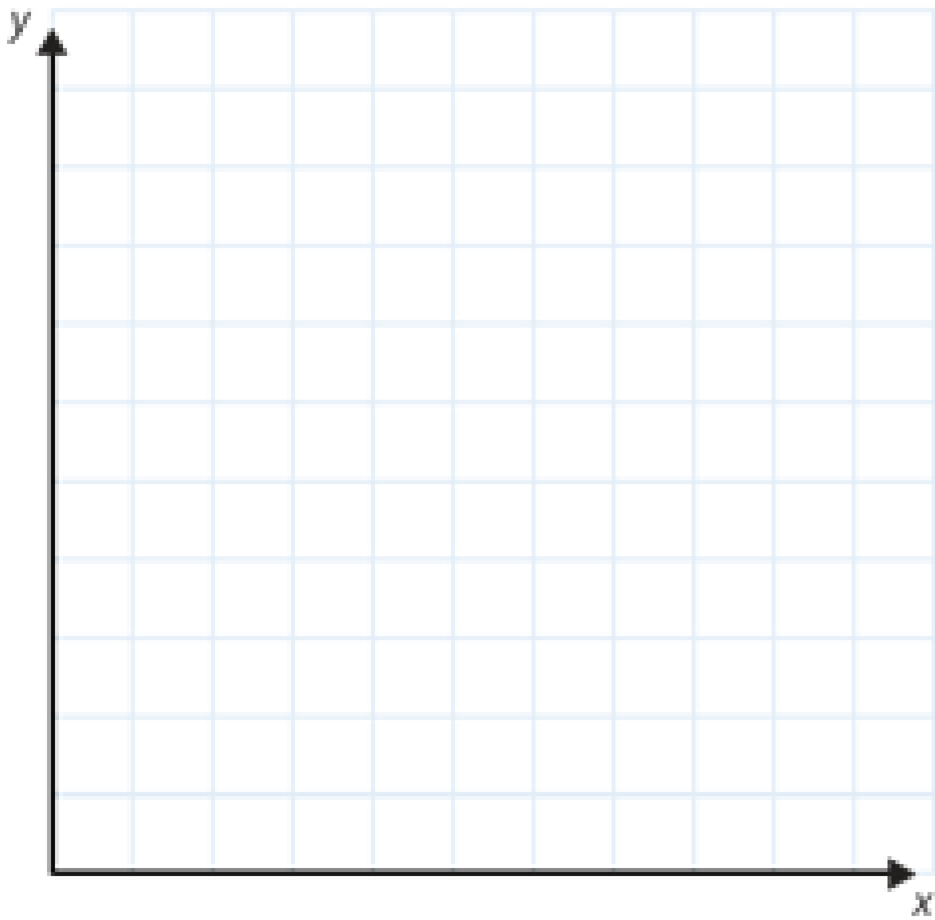
11. Assuming the pattern continues in the table above, write an equation to describe the relationship between x and y.
12. Create a graph to represent the relationship between the number of gallons purchased, x, and the total cost, y. Use the blank axes below to create your graph.



13. Assuming the pattern continues, determine the cost of buying 3 gallons of gasoline at Gary's Gas Station.

Franklin has a \$10 bus card. Each time he rides the bus, \$2 is deducted from his card. The equation $y = 10 - 2x$ represents the relationship between x, the number of bus rides Franklin takes and y, the number of dollars that are left on his card.

14. Create a table to show how many dollars will be left on Franklin's bus card after he has used it for a total of 0, 1, 2, and 3 bus rides.
15. Create a graph that represents the relationship between the total number of bus rides Franklin takes, x, and the number of dollars left on the card, y. Use the blank axes below to create your graph.



16. If Franklin takes a total of 4 bus rides, how many dollars will be left on his bus card?

Review (Answers)

To see the Review answers, open this PDF file and look for section 7.19.

1.20 References

1. CK-12, Larame Spence. Desmos Graphing Calculator .
2. CK-12, Larame Spence. Desmos Graphing Calculator .