


Problem of the Week
Problem C and Solution
The Hat Race

Problem

Frankie, Chrystal, and Sam have just competed in the POTW's 10^{th} annual Toboggan Race.

Frankie, Chrystal, and Sam each finished in first, second, or third in the toboggan race. There were no ties.

Each person also wore a different colour hat. One wore a red hat, one wore a green hat, and one wore a purple hat.

Using the following clues, determine who placed first, second and third, and which hat each person wore.

1. Chrystal was faster than Sam.
2. Sam did not wear the purple hat since she does not like purple and she did not finish before Frankie.
3. The person who wore the red hat was faster than the person who wore the green hat.
4. Chrystal did not wear the red hat and Frankie did not wear the green hat.
5. The person who came in first did not wear the purple hat.

Solution

Answer

We will present the answer first for those who want to check their work. The solution that follows represents one possible approach to arriving at a correct set of conclusions.

Frankie wore the red hat and finished in first.

Chrystal wore the purple hat and finished in second.

Sam wore the green hat and finished in third.

Solution

In our solution, we will we go through each clue and update the table based on the information in the clue. We will put an X in a cell if the combination indicated by the row and column for that cell is not possible, or a ✓ if it must be true.

From clue (1), since Chrystal was faster than Sam, we know that Sam could not have finished first and that Chrystal did not finish third. We can therefore put an X in the cells corresponding to Sam in first and Chrystal in third.

From clue (2), we can put an X in the cells corresponding to Sam wearing the purple hat and also to Frankie finishing in third since Frankie finished before Sam. The table is updated to the right.

	Red Hat	Green Hat	Purple Hat	First	Second	Third
Frankie						X
Chrystal						X
Sam			X	X		
First						
Second						
Third						

From the previous table we see that neither Frankie nor Chrystal finished third. Therefore, Sam must have finished third. We can add a ✓ to the corresponding cell in the table.

Since Sam finished third, she could not have also finished in second. So, we can add an X to the corresponding cell in the table. The table is updated to the right.

	Red Hat	Green Hat	Purple Hat	First	Second	Third
Frankie						X
Chrystal						X
Sam			X	X	X	✓
First						
Second						
Third						

From clue (3), we know that the person who wore the green hat did not finish first and that the person who wore the red hat did not finish third. We can therefore put an X in the appropriate cells.

Since we now know that Sam came in third, this clue also tells us that Sam is not the person who wore the red hat. We can put an X in the corresponding cell. The table is updated to the right.

	Red Hat	Green Hat	Purple Hat	First	Second	Third
Frankie						X
Chrystal						X
Sam	X		X	X	X	✓
First		X				
Second						
Third	X					

From the previous table we see that Sam did not wear the red hat or the purple hat. Therefore, Sam must have worn the green hat.

Since Sam finished third, this also tells us that the person wearing the green hat finished third.

We can add a ✓ to the corresponding cells in the table. The table is updated to the right.

	Red Hat	Green Hat	Purple Hat	First	Second	Third
Frankie						X
Chrystal						X
Sam	X	✓	X	X	X	✓
First		X				
Second						
Third	X	✓				

Since the people wear different hats, we now know that Chrystal and Frankie did not wear the green hat, and so we can add X's to the corresponding cells in the table.

Since we know that the person in the green hat finished third, they also could not have finished second. Similarly, the person in the purple hat could not have finished third. We can add X's to the corresponding cells in the table.

The table is updated to the right.

	Red Hat	Green Hat	Purple Hat	First	Second	Third
Frankie		X				X
Chrystal		X				X
Sam	X	✓	X	X	X	✓
First		X				
Second		X				
Third	X	✓	X			

From clue (4), we can add an X to the cell corresponding to Chrystal wearing the red hat.

We know that neither Chrystal nor Sam wore the red hat. Therefore, Frankie must have worn the red hat. Similarly, Chrystal did not wear the red hat or the green hat. Therefore, Chrystal must have worn the purple hat.

We can add the corresponding ✓'s and X's. The table is updated to the right.

	Red Hat	Green Hat	Purple Hat	First	Second	Third
Frankie	✓	X	X			X
Chrystal	X	X	✓			X
Sam	X	✓	X	X	X	✓
First		X				
Second		X				
Third	X	✓	X			

From clue (5), we know that the person who wore the purple hat did not come in first. Since Chrystal wore the purple hat, she did not come in first. This means that she must have finished second and it then follows that Frankie finished in first.

We can add the corresponding ✓'s and X's. The table is updated as shown to the right.

	Red Hat	Green Hat	Purple Hat	First	Second	Third
Frankie	✓	X	X	✓	X	X
Chrystal	X	X	✓	X	✓	X
Sam	X	✓	X	X	X	✓
First		X				
Second		X				
Third	X	✓	X			

In the previous table the top three rows have been completed. We can use this information to fill in the four remaining empty cells in the bottom three rows of the table.

The completed table is shown to the right.

	Red Hat	Green Hat	Purple Hat	First	Second	Third
Frankie	✓	X	X	✓	X	X
Chrystal	X	X	✓	X	✓	X
Sam	X	✓	X	X	X	✓
First	✓	X	X			
Second	X	X	✓			
Third	X	✓	X			

From the completed table, we see that:

Frankie wore the red hat and finished in first.

Chrystal wore the purple hat and finished in second.

Sam wore the green hat and finished in third.