

3.3

Adding and Subtracting Decimals

Focus

Add and subtract decimals to thousandths.

When you go to the theatre to see a movie, your attendance and how much you paid to see the movie are entered in a database.

Data are collected from theatres all across Canada and the United States.

Movie studios use these data to help predict how much money the movie will earn.



Explore



Shrek 2 was one of the highest-earning movies of 2004.

The table shows how much money *Shrek 2* earned in Canada and the United States for the first week it played in theatres.

Studios record the earnings in millions of US dollars.

➤ Estimate first.

Then find the combined earnings on:

- the first 2 days
- Saturday and Sunday
- all 7 days

➤ Estimate first.

Then find the difference in earnings on:

- Thursday and Friday
- Saturday and Sunday
- Sunday and Monday
- the days with the greatest and the least earnings

Date	Earnings (US\$ Millions)
Wednesday, May 19	11.786
Thursday, May 20	9.159
Friday, May 21	28.340
Saturday, May 22	44.797
Sunday, May 23	34.901
Monday, May 24	11.512
Tuesday, May 25	8.023



Reflect & Share

Share your results with another pair of classmates.

Discuss the strategies you used to estimate and to find the sums and differences.

Why do you think the earnings on 3 of the days are so much higher? Explain.

Connect

When we add or subtract decimals, we estimate if we do not need an exact answer.
We also estimate to check the answer is reasonable.

Example

Ephram is a long-distance runner. His practice distances for 5 days last week are shown in the table.

- a) How far did Ephram run in 5 days last week?
- b) How much farther did Ephram run on Tuesday than on Thursday?

Day	Distance (km)
Monday	8.85
Tuesday	12.25
Wednesday	10.9
Thursday	9.65
Friday	14.4

A Solution

- a) $8.85 + 12.25 + 10.9 + 9.65 + 14.4$

Use front-end estimation.

Add the whole-number part of each decimal.

Think: $8 + 12 + 10 + 9 + 14 = 53$

Ephram ran about 53 km.

Add. Write each number with the same number of decimal places.

Use zeros as placeholders: 8.85, 12.25, 10.90, 9.65, 14.40

Record the numbers without the decimal points.

Add as you would whole numbers.

$$\begin{array}{r} 231 \\ 885 \\ 1225 \\ 1090 \\ 965 \\ + 1440 \\ \hline 5605 \end{array}$$

Since the estimate is 53 km, place the decimal point after the first 2 digits; that is, between the 6 and the 0.

Ephram ran 56.05 km.

- b) Ephram ran 12.25 km on Tuesday and 9.65 km on Thursday.

Estimate.

$$12.25 - 9.65$$

Think: $12 - 9 = 3$

Ephram ran about 3 km farther on Tuesday.



Subtract. Align the numbers.

Subtract as you would whole numbers.

$$\begin{array}{r} 12.25 \\ - 9.65 \\ \hline 2.60 \end{array}$$

2.6 is close to the estimate 3, so the answer is reasonable.

Ephram ran 2.6 km farther on Tuesday than on Thursday.

Practice

- Use front-end estimation to estimate each sum or difference.
 - $2.876 - 0.975$
 - $71.382 + 6.357$
 - $125.12 + 37.84$
 - $9.7 - 1.36$
- The tallest building in the world is the Taipei 101 in Taiwan. Its height is 0.509 km. The tallest building in North America is the Sears Tower in Chicago, USA. Its height is 0.442 km. What is the difference in the heights of the buildings?
- Four classes of students from Mackenzie School are planning a field trip. The total cost of the trip is \$1067.50. To date, the classes have raised: \$192.18, \$212.05, \$231.24, \$183.77
 - How much money have the classes raised so far?
 - How much more money do the classes need to raise in total?Show your work.
- Assessment Focus** A baker wants to make 3 different kinds of chocolate chip cookies. The recipes call for 2.75 kg, 4.4 kg, and 5.55 kg of chocolate chips. The baker has 10.5 kg of chocolate chips.
 - How many kilograms of chocolate chips does the baker need?
Estimate to check your answer is reasonable.
 - Does the baker have enough chocolate chips to make the cookies?
How do you know?
 - The baker wants to follow the recipes exactly.
If your answer to part b is no, how many more kilograms of chocolate chips are needed? If your answer to part b is yes, how many kilograms of chocolate chips will the baker have left over?



5. Estimate, then calculate, the sum below.

Explain how you estimated.

$$46.71 + 3.9 + 0.875$$

6. The Robb family and the Chan family have similar homes.

The Robb family sets its thermostat to 20°C during the winter months.

Its monthly heating bills were: \$171.23, \$134.35, and \$123.21

The Chan family used a programmable thermostat to lower the temperature at night, and during the day when the family was out.

The Chan family's monthly heating bills were: \$134.25, \$103.27, and \$98.66

- How much money did each family pay to heat its home during the winter months?
- How much more money did the Robb family pay?
Estimate to check your answer is reasonable.
- What other things could a family do to reduce its heating costs?



7. Find two numbers with a difference of 151.297.

8. Use each of the digits from 0 to 7 once to make this addition true.

Find as many different answers as you can.

$$\begin{array}{r} \square.\square\square\square \\ + \square.\square\square\square \\ \hline 5.788 \end{array}$$

9. A student subtracted 0.373 from 4.81 and got the difference 0.108.

- What mistake did the student make?
- What is the correct answer?

10. Two 4-digit numbers were added. Their sum was 3.3.

What could the numbers have been?

Find as many different answers as you can. Show your work.

11. **Take It Further** Find each pattern rule. Explain how you found it.

- 2.09, 2.13, 2.17, 2.21, ...
- 5.635, 5.385, 5.135, 4.885, ...

Reflect

How did your knowledge of estimation help you in this lesson?

3.4

Multiplying Decimals

Focus

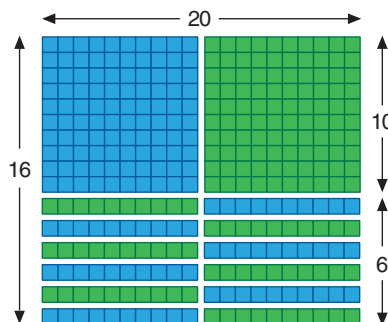
Use Base Ten Blocks, paper and pencil, and calculators to multiply decimals.

Recall how to multiply 2 whole numbers using Base Ten Blocks.

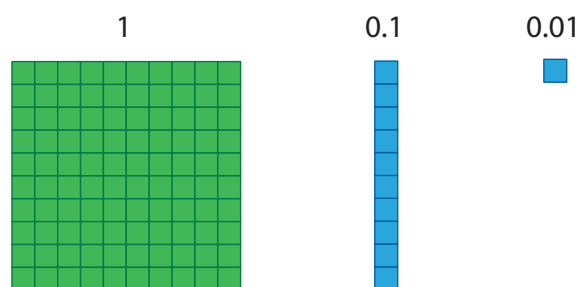
This picture shows the product:

$$20 \times 16 = 100 + 100 + 60 + 60 = 320$$

We can also use Base Ten Blocks to multiply 2 decimals.



Let the flat represent 1, the rod represent 0.1, and the small cube represent 0.01.



Explore



You will need Base Ten Blocks and grid paper.
Use Base Ten Blocks to model a rectangular patio with area greater than 4 m² and less than 6 m².
Let the side length of the flat represent 1 m.
How many different patios can you model?
Record your designs on grid paper.

Reflect & Share

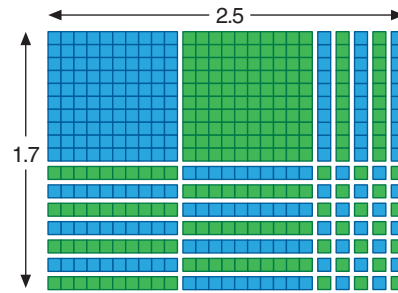
Compare your designs with those of another pair of classmates.
Did you have any designs the same? Explain.
Explain how your designs show the area of the patio.



Connect

A rectangular park measures 1.7 km by 2.5 km.
Here are 2 ways to find the area of the park.

- Use Base Ten Blocks.
Build a rectangle with length 2.5 and width 1.7.
Count the blocks in the rectangle.
There are 2 flats: $2 \times 1 = 2$
There are 19 rods: $19 \times 0.1 = 1.9$
There are 35 small cubes: $35 \times 0.01 = 0.35$
The total area is: $2 + 1.9 + 0.35 = 4.25$
The total area of the park is 4.25 km^2 .



- Use the method for multiplying 2 whole numbers.
The area, in square kilometres, is 1.7×2.5 .

$$\begin{array}{r} 17 \times 25 \\ \times 25 \\ \hline 85 \\ 340 \\ \hline 425 \end{array}$$

1.7×2.5
Think: $1 \times 2 = 2$
So, 1.7×2.5 is about 2.
Place the decimal point
between the 4 and the 2.

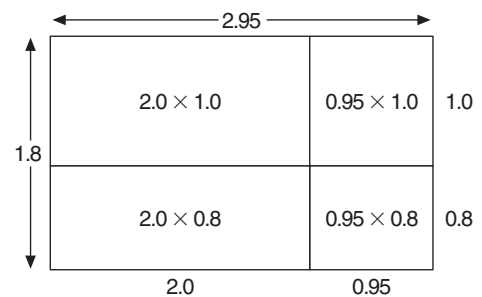
Using front-end estimation to place the decimal point, $1.7 \times 2.5 = 4.25$.
The area of the park is 4.25 km^2 .

Example

At the Farmers' Market, 1 kg of grapes costs \$2.95.
How much would 1.8 kg of grapes cost?

A Solution

1 kg of grapes costs \$2.95.
So, 1.8 kg would cost: $\$2.95 \times 1.8$
Use a rectangle model.



$$\begin{aligned} 2.95 \times 1.8 &= (2.0 \times 1.0) + (0.95 \times 1.0) + (2.0 \times 0.8) + (0.95 \times 0.8) \\ &= (2 \times 1) + (0.95 \times 1) + (2 \times 0.8) + (0.95 \times 0.8) \\ &= 2 + 0.95 + 1.6 + 0.76 \\ &= 5.31 \end{aligned}$$

1.8 kg of grapes would cost \$5.31.

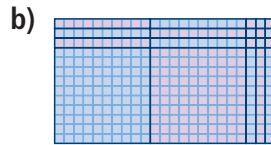
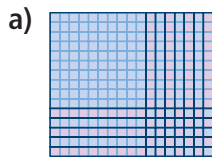
Another Strategy
We could use a calculator
to multiply.

Use a calculator when the multiplier has more than 2 digits.

Practice

1. Write the product that each picture represents.

Each small square represents 0.01.



2. Use Base Ten Blocks to find each product.

Record your work on grid paper.

a) 2.6×1.5

b) 2.3×0.4

c) 0.8×0.7

3. Choose one part from question 2.

Explain how the Base Ten Blocks show the product.

4. Multiply. Use a rectangle model.

a) 4.2×3.7

b) 8.9×0.3

c) 0.6×0.9

5. A rectangular plot of land measures 30.5 m by 5.3 m.

What is the area of the plot?

Estimate to check your answer is reasonable.



6. Multiply. Describe any patterns you see.

a) 8.36×10

b) 8.36×0.1

8.36×100

8.36×0.01

8.36×1000

8.36×0.001

$8.36 \times 10\,000$

8.36×0.0001

7. **Assessment Focus** An area rug is rectangular.

Its dimensions are 3.4 m by 2.7 m.

Show different strategies you can use to find the area of the rug.

Which strategy is best? Justify your answer.



8. Multiply.

a) 2.7×4.786

b) 12.52×13.923

c) 0.986×1.352

Explain how you can check your answers.



- 9.** The fuel consumption estimates of Josie's car are:

City: 21.2 km/L Highway: 23.3 km/L

The car's gas tank holds 40.2 L of fuel.

- How far could Josie drive on a full tank of gas on the highway before she runs out of fuel?
- How far could she drive on a full tank of gas in the city?
What assumptions did you make?

- 10.** Find the cost of each item at the Farmers' Market.

Which strategy will you use? Justify your choice.

- 2.56 kg of apples at \$0.95/kg
- 10.5 kg of potatoes at \$1.19/kg
- 0.25 kg of herbs at \$2.48/kg

- 11.** The product of 2 decimals is 0.36.

What might the decimals be?

Find as many answers as you can.



- 12.** a) Multiply 18×12 .

- b) Use only the result from part a and estimation.

Find each product.

i) 1.8×12

ii) 18×0.12

iii) 0.18×12

iv) 0.18×0.12

Explain your strategies.



- 13. Take It Further**

- a) Multiply.

i) 6.3×1.8

ii) 0.37×0.26

iii) 3.52×2.4

iv) 1.234×0.9

- b) Look at the questions and products in part a.

What patterns do you see in the numbers of decimal places in the question and the product?

How could you use this pattern to place the decimal point in a product without estimating?

- c) Multiply: 2.6×3.5

Does the pattern from part b hold true?

If your answer is no, explain why not.

Reflect

When you multiply 2 decimals, how do you know where to place the decimal point in the product? Use examples to explain.

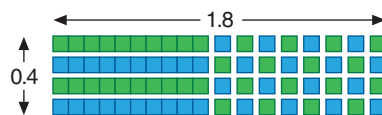
3.5 Dividing Decimals

Focus Use Base Ten Blocks, paper and pencil, and calculators to divide decimals.

Recall how you used Base Ten Blocks to multiply:

Since multiplication and division are related, we can also use Base Ten Blocks to divide.

Which division sentences could you write for this diagram?



$$1.8 \times 0.4 = 0.72$$

Explore



You will need Base Ten Blocks and grid paper. Marius bought 1.44 m of ribbon for his craft project. He needs to cut the ribbon into 0.6-m lengths. How many 0.6-m lengths can he cut? Use Base Ten Blocks to find out. Record your work on grid paper.



Reflect & Share

Compare your solution with that of another pair of classmates. What was your strategy? How could you use division of whole numbers to check your answer?

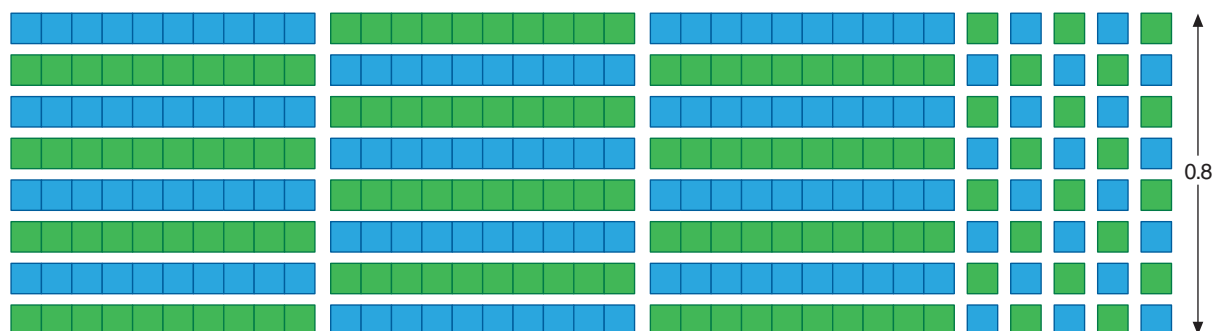
Connect

Jan bought 2.8 m of framing to make picture frames. Each picture needs 0.8 m of frame. How many frames can Jan make? How much framing material is left over?

Use Base Ten Blocks to divide: $2.8 \div 0.8$



Make a rectangle with area 2.8 and width 0.8.



The length of the rectangle is 3.5.

So, Jan can make 3 frames.

3 frames use: $3 \times 0.8 \text{ m} = 2.4 \text{ m}$

So, the framing material left is: $2.8 \text{ m} - 2.4 \text{ m} = 0.4 \text{ m}$

Sometimes when we divide 2 decimals, the quotient is not a terminating decimal.

Then we can use paper and pencil.

Example

Divide: $52.1 \div 0.9$

A Solution

Estimate first: $52.1 \div 0.9$

Write each decimal to the nearest whole number, then divide.

$$52 \div 1 = 52$$

So, $52.1 \div 0.9$ is about 52.

52.1 is closer to 52.

0.9 is closer to 1.

Divide as you would whole numbers.

$$521 \div 9$$

$$\begin{array}{r} \text{divisor} \rightarrow 9 \overline{) 52100} \\ \underline{45} \\ 71 \\ \underline{63} \\ 80 \\ \underline{72} \\ 80 \\ \underline{72} \\ 8 \end{array}$$

← quotient
← dividend

Divide until the quotient has 2 more digits than the estimate. Then we can write the quotient to the nearest tenth.

If the quotient is not exact, write zeros in the dividend, then continue to divide.

Since the estimate has 2 digits, divide until there are 4 digits in the quotient.

Since the estimate was 52, place the decimal point

so the quotient is close to 52: $52.1 \div 0.9 \doteq 57.88$

In the question, the dividend and divisor were given to the nearest tenth.

So, we write the quotient to the nearest tenth.

$52.1 \div 0.9 \doteq 57.90$, or 57.9

57.88 is closer to 57.90 than to 57.80.

We can use a calculator when the divisor has more than 1 digit.

Practice

1. Use Base Ten Blocks to divide. Record your work on grid paper.

a) $0.8 \div 0.1$ b) $1.2 \div 0.3$ c) $2.7 \div 0.6$ d) $2.2 \div 0.4$



2. Divide. Describe any patterns you see.

a) $124.5 \div 10$	b) $124.5 \div 0.1$
$124.5 \div 100$	$124.5 \div 0.01$
$124.5 \div 1000$	$124.5 \div 0.001$
$124.5 \div 10\,000$	$124.5 \div 0.0001$

3. Why do all these division statements have 6 as the answer?

a) $30 \div 5$ b) $3.0 \div 0.5$ c) $0.3 \div 0.05$ d) $300 \div 50$

Which one is easiest to calculate? Explain.

4. Estimate to choose the correct quotient for each division question.

Question	Possible Quotients		
a) $59.5 \div 5$	119	11.9	1.19
b) $195.3 \div 0.2$	9765	976.5	97.65
c) $31.32 \div 0.8$	3915	391.5	39.15

5. Use paper and pencil to divide.

a) $1.5 \div 0.6$ b) $2.24 \div 0.7$ c) $1.28 \div 0.8$ d) $2.16 \div 0.9$



6. Divide. Write each quotient to the nearest tenth.

Use front-end estimation to check your answer is reasonable.

a) $8.36 \div 2.4$ b) $1.98 \div 1.3$ c) $27.82 \div 3.9$ d) $130.4 \div 5.4$

7. A toonie is approximately 0.2 cm thick.

How many toonies are in a stack of toonies 17.4 cm high?

8. The area of a large rectangular flowerbed is 22.32 m^2 .
The width is 0.8 m . What is the length?
9. A 0.4-kg bag of oranges costs $\$1.34$.
- Estimate. About how much does 1 kg of oranges cost?
 - What is the actual cost of 1 kg of oranges?
How do you know your answer is reasonable?
 - Suppose you spent $\$10$ on oranges.
What mass of oranges did you buy?

10. **Assessment Focus** Alex finds a remnant of landscaping fabric at a garden store. The fabric is the standard width, with length 9.88 m . Alex needs fourteen 0.8-m pieces for a garden patio.
- How many 0.8-m pieces can Alex cut from the remnant?
What assumptions did you make?
 - Will Alex have all the fabric he needs? Why or why not?
 - If your answer to part b is no, how much more fabric does Alex need?
 - Alex redesigns his patio so that he needs fourteen 0.7-m pieces of fabric.
Will the remnant be enough fabric? Explain.



11. The quotient of two decimals is 0.12 . What might the decimals be?
Write as many different possible decimal pairs as you can.
12. Last week, Alicia worked 37.5 h . She earned $\$346.88$.
How much money did Alicia earn per hour?
Why is the answer different from the number in the calculator display?

13. The question $237 \div 7$ does not have an exact quotient.
The first five digits of the quotient are 33857 .
The decimal point has been omitted. Use only this information and estimation.
Write an approximate quotient for each question.
Justify each answer.
- $237 \div 0.7$
 - $2.37 \div 0.07$
 - $23.7 \div 7$
 - $2370 \div 70$

Reflect

Talk to a partner. Tell how you can find $1.372 \div 0.7$ by dividing by 7 .
Why does this work?

3.6

Order of Operations with Decimals

Focus

Use the order of operations to evaluate expressions.

Explore



How many different ways can you find the answer for this expression?

$$6 \times 15.9 + 36.4 \div 4$$

Show your work for each answer.

Reflect & Share

Compare your answers with those of another pair of classmates.

Which solution do you think is correct? Explain your reasoning.

Connect

To make sure everyone gets the same answer for a given expression, we add, subtract, multiply, and divide in this order:

- Do the operations in brackets first.
- Then divide and multiply, in order, from left to right.
- Then add and subtract, in order, from left to right.

When we find the answer to an expression, we *evaluate*.

We use the same order of operations for decimals as for whole numbers.

Example

Evaluate: $12.376 \div (4.75 + 1.2) + 2.45 \times 0.2 - 1.84$

Use a calculator when you need to.

A Solution

$$12.376 \div (4.75 + 1.2) + 2.45 \times 0.2 - 1.84 \quad \text{Calculate in brackets.}$$

$$= 12.376 \div 5.95 + 2.45 \times 0.2 - 1.84 \quad \text{Multiply and divide from left to right.}$$

$$= 2.08 + 0.49 - 1.84 \quad \text{Add and subtract from left to right.}$$

$$= 2.57 - 1.84$$

$$= 0.73$$

Many calculators follow the order of operations.

To see whether your calculator does, enter: $12.4 \times 2.2 - 15.2 \div 4$

If your answer is 23.48, your calculator follows the order of operations.

Practice

1. Evaluate.

a) $4.6 + 5.1 - 3.2$ b) $8 - 3.6 \div 2$ c) $46.4 - 10.8 \times 3$ d) $85.6 \div 0.4 \times 7$

2. Evaluate.

a) $(46.78 - 23.58) \times 2.5$ b) $(98.5 + 7) \div 0.5$ c) $7.2 \div (2.4 - 1.8)$

3. Evaluate.

a) $9.8 - 3.2 \div 0.4 + 2.6$ b) $(9.8 - 3.2) \div (0.4 + 2.6)$

Explain why the answers are different.



4. Evaluate.

a) $1.35 + (5 \times 4.9 \div 0.07) - 2.7 \times 2.1$ b) $9.035 \times 5.2 - 4.32 \times 6.7$
c) $2.368 \div 0.016 + 16.575 \div 1.105$ d) $0.38 + 16.2 \times (2.1 + 4.7) + 21 \div 3.5$

5. **Assessment Focus** Ioana, Aida, and Norman got different answers

for this problem: $12 \times (4.8 \div 0.3) - 3.64 \times 3.5$

Ioana's answer was 39.12, Aida's answer was 179.26,

and Norman's answer was 659.26.

a) Which student had the correct answer?

How do you know?

b) Show and explain how the other two students got their answers.

Where did they go wrong?



6. Evaluate. Show all steps:

$0.38 + 16.2 \times (2.1 - 1.2) + 21 \div 0.8$

7. **Take It Further** Use at least 4 of the numbers 0.1, 0.2, 0.3, 0.4, 0.5, 0.6, 0.7, 0.8 and 0.9, and any operations or brackets to make each whole number from 1 to 5.

Reflect

Why do we need to agree on an order of operations?

Mid-Unit Review

LESSON

Use a calculator when you need to.

3.1 1. a) Write each fraction as a decimal.

i) $\frac{1}{33}$ ii) $\frac{2}{33}$ iii) $\frac{3}{33}$

b) Describe the pattern in your answers to part a.

c) Use your pattern to predict the fraction form of each decimal.

i) $0.\overline{15}$ ii) $0.\overline{36}$

2. Write each fraction as a decimal.

Identify the decimals as repeating or terminating.

a) $\frac{1}{8}$ b) $\frac{3}{5}$

c) $\frac{2}{3}$ d) $\frac{7}{13}$

3. Write each decimal as a fraction.

a) 0.2 b) $0.\overline{8}$

c) 0.005 d) $0.\overline{23}$

3.2 4. Order each set of numbers from least to greatest. Use a different method for each part.

a) $2\frac{1}{4}, \frac{11}{6}, \frac{8}{3}, 2$ b) $3.5, \frac{23}{8}, 1\frac{3}{4}$

c) $1.75, \frac{13}{10}, \frac{9}{5}, 1\frac{3}{5}, 1$

5. Find a number between each pair of numbers. Which strategy did you use each time?

a) $\frac{4}{3}, \frac{5}{3}$ b) $2\frac{3}{8}, \frac{5}{2}$ c) $1.4, \frac{8}{5}$

3.3 6. Use front-end estimation to place the decimal point in each answer.

a) $32.47 - 6.75 = 2572$

b) $118.234 + 19.287 = 137521$

c) $17.9 - 0.8 = 171$

7. Winsome is being trained as a guide dog for a blind person.

At birth, she had a mass of 0.475 kg.

At 6 weeks, her mass was 4.06 kg.

From 6 weeks to 12 weeks, she gained 5.19 kg.

a) By how much did Winsome's mass change from birth to 6 weeks?

b) What was her mass at 12 weeks?

3.4 8. Estimate to place the decimal point in each product.

Show your estimation strategy.

a) $9.3 \times 0.8 = 744$

b) $3.62 \times 1.3 = 4706$

c) $11.25 \times 5.24 = 5895$

9. A rectangular park has dimensions 2.84 km by 3.5 km.

What is the area of the park?

3.5 10. When you divide 15.4 by 2, the quotient is 7.7.

When you divide 1.54 by 0.2, the quotient is 7.7.

Explain why the quotients are the same.

3.6 11. Evaluate.

a) $5.9 + 3.7 \times 2.8$

b) $12.625 \times (1.873 + 2.127)$

c) $2.1 \div 0.75 + 6.38 \times 2.45$