We can use symbols to represent a pattern.

Explore

Tehya won some money in a competition.
She has two choices as to how she gets paid.
Choice 1: $20 per week for one year
Choice 2: $400 cash now plus $12 per week for one year

Which method would pay Tehya more money?
For what reasons might Tehya choose each method of payment?

Reflect & Share

Work with another pair of classmates.
For each choice, describe a rule you can use to calculate the total money
Tehya has received at any time during the year.

Connect

We can use a variable to represent a number in an expression.
For example, we know there are 100 cm in 1 m.

1 m = 100 cm

We can write 1 × 100 cm in 1 m.
There are 2 × 100 cm in 2 m.
There are 3 × 100 cm in 3 m.

> Recall that a variable is a letter, such as n, that represents a quantity that can vary.

To write an expression for the number of centimetres in any number of metres,
we say there are n × 100 cm in n metres.
n is a variable.
n represents any number we choose.

We can use any letter, such as n or x, as a variable.
The expression n × 100 is written as $100n$.
$100n$ is an **algebraic expression**.

> Variables are written in italics so they are not confused with units of measurement.

Here are some other algebraic expressions, and their meanings.
In each case, n represents the number.

- Three more than a number: $3 + n$ or $n + 3$
- Seven times a number: $7n$
- Eight less than a number: $n - 8$
- A number divided by 20: $\frac{n}{20}$

> $7n$ means $7 \times n$.

When we replace a variable with a number in an algebraic expression, we *evaluate* the expression. That is, we find the value of the expression for a particular value of the variable.

Example

Write each algebraic expression in words.
Then evaluate for the value of the variable given.

a) $5k + 2$ for $k = 3$ **b)** $32 - \frac{x}{4}$ for $x = 20$

A Solution

a) $5k + 2$ means 5 times a number, then add 2.
   Replace k with 3 in the expression $5k + 2$.
   Then use the order of operations.
   $5k + 2 = 5 \times 3 + 2$ Multiply first.
   $\qquad\quad = 15 + 2$ Add.
   $\qquad\quad = 17$

b) $32 - \frac{x}{4}$ means 32 minus a number divided by 4.
   Replace x with 20 in the expression $32 - \frac{x}{4}$.
   Then use the order of operations.
   $32 - \frac{x}{4} = 32 - \frac{20}{4}$ Divide first.
   $\qquad\quad = 32 - 5$ Subtract.
   $\qquad\quad = 27$

> $\frac{x}{4}$ means $x \div 4$.

In the expression $5k + 2$,

- 5 is the **numerical coefficient** of the variable.
- 2 is the **constant term**.
- k is the *variable*.
   The variable represents any number in a set of numbers.

1. Identify the numerical coefficient, the variable,
and the constant term in each algebraic expression.
   a) $3x + 2$ **b)** $5n$ **c)** $w + 3$ **d)** $2p + 4$

2. An algebraic expression has variable p, numerical coefficient 7,
and constant term 9.
Write as many different algebraic expressions as you can
that fit this description.

3. Write an algebraic expression for each phrase.
   a) six more than a number
   b) a number multiplied by eight
   c) a number decreased by six
   d) a number divided by four

4. A person earns $4 for each hour he spends baby-sitting.
   a) Find the money earned for each time.
      i) 5 h   **ii)** 8 h
   b) Write an algebraic expression you could use
      to find the money earned in t hours.

5. Write an algebraic expression for each sentence.
   a) Double a number and add three.
   b) Subtract five from a number, then multiply by two.
   c) Divide a number by seven, then add six.
   d) A number is subtracted from twenty-eight.
   e) Twenty-eight is subtracted from a number.

6. a) Write an algebraic expression for each phrase.
      i) four more than a number
      ii) a number added to four
      iii) four less than a number
      iv) a number subtracted from four
   b) How are the expressions in part a alike?
      How are they different?

7. Evaluate each expression by replacing x with 4.

a) $x + 5$ b) $3x$ c) $2x - 1$
d) $\frac{x}{2}$ e) $3x + 1$ f) $20 - 2x$

8. Evaluate each expression by replacing z with 7.

a) $z + 12$ b) $10 - z$ c) $5z$
d) $3z - 3$ e) $35 - 2z$ f) $3 + \frac{z}{7}$

9. **Assessment Focus** Jason works at a local
fish and chips restaurant.
He earns $7/h during the week, and $9/h on
the weekend.

a) Jason works 8 h during the week and
12 h on the weekend.
Write an expression for his earnings.

b) Jason works x hours during the week and
5 h on the weekend.
Write an expression for his earnings.

c) Jason needs $115 to buy sports equipment.
He worked 5 h on the weekend.
How many hours does Jason have to work
during the week to have the money he needs?

10. **Take It Further** A value of n is substituted in each expression
to get the number in the box.
Find each value of n.

a) $5n$ `30` b) $3n - 1$ `11`
c) $4n + 7$ `15` d) $5n - 4$ `11`
e) $4 + 6n$ `40` f) $\frac{n}{8}$ `5`

Reflect

Explain why it is important to use the order of operations
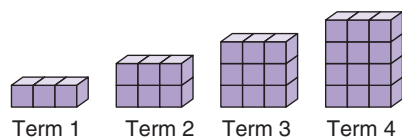when evaluating an algebraic expression.
Use an example in your explanation.

Here is a pattern made from linking cubes.

Term 1   Term 2   Term 3   Term 4

A pattern rule is: Start at 3. Add 3 each time.
This rule relates each term to the term that comes before it.

We can also describe this pattern using the term number.

Term Number	1	2	3	4
Term	3	6	9	12

How does each term relate to the term number?

Explore

On Enviro-Challenge Day, Grade 7 classes compete
to see which class can collect the most garbage.

Each student in Ms. Thomson's class pledges
to pick up 6 pieces of garbage.
➤ How many pieces of garbage will be picked up when
   the number of students is 5? 10? 15? 20? 25? 30?
➤ What pattern do you see in the numbers of pieces
   of garbage?
➤ Write a rule to find how many pieces of garbage
   will be picked up, when you know the number of students.
➤ Write an algebraic expression for the number of pieces
   of garbage picked up by *n* students.

Reflect & Share

Share your work with another pair of classmates.
Find the number of pieces of garbage picked up by 35 students.
How can you do this using the pattern?
Using the rule? Using the algebraic expression?

Miss Jackson's class pledges to pick up a total of 10 more pieces of garbage than Ms. Thomson's class.
Here are the numbers of pieces of garbage picked up by different numbers of students.

Number of students	2	4	6	8	10	12
Number of pieces of garbage picked up by Ms. Thomson's class	12	24	36	48	60	72
Number of pieces of garbage picked up by Miss Jackson's class	22	34	46	58	70	82

Pieces of garbage picked up by Miss Jackson's class $=$ 10 $+$ Pieces of garbage picked up by Ms. Thomson's class

Let n represent the number of students who pick up garbage in Ms. Thomson's class.
Then the number of pieces of garbage picked up by Ms. Thomson's class is $6n$.
And, the number of pieces of garbage picked up by Miss Jackson's class is $10 + 6n$.
The number of pieces of garbage is *related* to the number of students.
When we compare or *relate* a variable to an expression that contains the variable, we have a **relation**.
That is, $10 + 6n$ is related to n.

> **Recall that 10 is the constant term.**

This is a relation.

Example

Mr. Prasad plans to hold a party for a group of his friends.
The cost of renting a room is $35.
The cost of food is $4 per person.
a) Write a relation for the cost of the party, in dollars, for n people.
b) How much will a party cost for 10 people? For 15 people?
c) How does the relation change if the cost of food doubles? How much more would a party for 10 people cost? How do you know the answer makes sense?

A Solution

a) The cost of renting a room is $35.

This does not depend on how many people come.

The cost of food is $4 per person.

If 5 people come, the cost of food in dollars is: $4 \times 5 = 20$

If n people come, the cost of food in dollars is: $4 \times n$, or $4n$

So, n is related to $35 + 4n$.

b) To find the cost for 10 people, substitute $n = 10$ into $35 + 4n$.

$$35 + 4n = 35 + 4(10)$$
$$= 35 + 40$$
$$= 75$$

The party will cost $75.

To find the cost for 15 people, substitute $n = 15$ into $35 + 4n$.

$$35 + 4n = 35 + 4(15)$$
$$= 35 + 60$$
$$= 95$$

The party will cost $95.

> $4(10)$ means 4×10.

c) If the cost of food doubles, Mr. Prasad will pay $8 per person.

If n people come, the cost for food, in dollars, is $8n$.

For n people, the cost of the party, in dollars, is now $35 + 8n$.

If 10 people come, the cost is now:

$$35 + 8n = 35 + 8(10)$$
$$= 35 + 80$$
$$= 115$$

The party will cost $115.

This is an increase of $115 - \$75 = \$40$.

The answer makes sense because the cost is now $4 more per person.

So, the extra cost for 10 people would be 4×10, or $40 more.

Math Link

History

The word "algebra" comes from the Arabic word "al-jabr." This word appeared in the title of one of the earliest algebra texts, written around the year 825 by al-Khwarizmi. He lived in what is now Uzbekistan.

1. **i)** For each number pattern, how is each term related to the term number?
   ii) Let *n* represent any term number. Write a relation for the term.

a)

Term Number	1	2	3	4	5	6
Term	2	4	6	8	10	12

b)

Term Number	1	2	3	4	5	6
Term	3	4	5	6	7	8

c)

Term Number	1	2	3	4	5	6
Term	8	16	24	32	40	48

d)

Term Number	1	2	3	4	5	6
Term	6	7	8	9	10	11

2. There are *n* students in a class. Write a relation for each statement.
   a) the total number of pencils, if each student has three pencils
   b) the total number of desks, if there are two more desks than students
   c) the total number of geoboards, if each pair of students shares one geoboard
   d) the total number of stickers, if each student gets four stickers and there are ten stickers left over

3. A person earns $10 for each hour worked.
   a) Write a relation for her earnings for *n* hours of work.
   b) How much does she earn for 30 h of work?

4. **a)** Write a relation for the perimeter of a square with side length *n* centimetres.
   b) What is the perimeter of a square with side length 12 cm?
   c) Suggest a situation that could be represented by each relation.
      i) 3*s* is related to *s*   **ii)** 8*t* is related to *t*
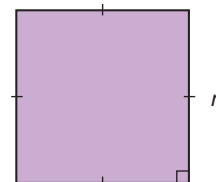
5. Suggest a real-life situation that could be represented by each relation.
   a) *n* + 5 is related to *n*   **b)** 15 + 2*p* is related to *p*
   c) 3*t* + 1 is related to *t*
   How do you know each situation fits the relation?

6. Koko is organizing an overnight camping trip. The cost to
rent a campsite is $20. The cost of food is $9 per person.
   a) How much will the trip cost if 5 people go? 10 people go?
   b) Write a relation for the cost of the trip when p people go.
   c) Suppose the cost of food doubles.
      Write a relation for the total cost of the trip for p people.
   d) Suppose the cost of the campsite doubles.
      Write a relation for the total cost of the trip for p people.
   e) Explain why using the variable p is helpful.

7. **Assessment Focus** A pizza with cheese and tomato toppings costs $8.00.
It costs $1 for each extra topping.
   a) Write a relation for the cost of a pizza with e extra toppings.
   b) What is the cost of a pizza with 5 extra toppings?
   c) On Tuesdays, the cost of the same pizza with cheese
      and tomato toppings is $5.00. Write a relation for the cost
      of a pizza with e extra toppings on Tuesdays.
   d) What is the cost of a pizza with 5 extra toppings on Tuesdays?
   e) How much is saved by buying the pizza on Tuesday?

8. Write a relation for the pattern rule for each number pattern.
Let n represent any term number.
   a) 4, 8, 12, 16, …   b) 7, 8, 9, 10, …   c) 0, 1, 2, 3, …

9. **Take It Further**
   i) For each number pattern, how is each term related to the term number?
   ii) Let n represent any term number. Write a relation for the term.

a)
Term Number	1	2	3	4	5	6
Term	3	5	7	9	11	13

b)
Term Number	3	4	5	6	7	8
Term	7	10	13	16	19	22

c)
Term Number	2	3	4	5	6	7
Term	5	9	13	17	21	25

Reflect

How did your knowledge of patterning help you in this lesson?