## 1.5

 Patterns and Relationships in TablesAn Input/Output machine represents a relation. Any Input number can be represented by $n$.

Suppose you input $n=8$.
What will the output be?
How is the output related to the input?


## Explore

Sketch an Input/Output machine like this one.


Write an algebraic expression to go in the machine.

- Use the numbers 1 to 6 as input.

Find the output for each Input number.
Record the input and output in a table like this.
> How is the output related to the input?

- Describe the pattern in the Output numbers.

| Input | Output |
| :---: | :---: |
| 1 |  |
| 2 |  |
| 3 |  |
| 4 |  |
| 5 |  |
| 6 |  |

## Reflect \& Share

Share your work with another pair of classmates.
Describe how you would find the next 3 Output numbers for your classmates' Input/Output machine.
How is the output related to the input?

## Connect

This Input/Output machine relates $n$ and $2 n+3$. To create a table of values, select a set of Input numbers. To get each Output number, multiply the Input number by 2 , then add 3.


$$
\begin{aligned}
& \text { When } n=1,2 n+3
\end{aligned}=2(1)+3 \text { } \begin{aligned}
& =2+3 \\
& =5 \\
\text { When } n=2,2 n+3 & =2(2)+3 \\
& =4+3 \\
& =7 \\
\text { When } n=3,2 n+3 & =2(3)+3 \\
& =6+3 \\
& =9
\end{aligned}
$$

Remember the order of operations. Multiply before adding.

| Input <br> $n$ | Output <br> $2 n+3$ |
| :---: | :---: |
| 1 | 5 |
| 2 | 7 |
| 3 | 9 |
| 4 | 11 |
| 5 | 13 |

and so on.
We used consecutive Input numbers.
The Output numbers form a pattern. They increase by 2 each time.
This is because the expression contains $2 n$, which means that the Input number is doubled.
When the Input number increases by 1 , the Output number increases by 2 .
The expression $2 n+3$ can also be written as $3+2 n$.
When a relation is represented as a table of values, we can write the relation using algebra.

## Example

Write the relation represented by this table.

| Input | Output |
| :---: | :---: |
| 1 | 2 |
| 2 | 5 |
| 3 | 8 |
| 4 | 11 |
| 5 | 14 |

## A Solution

Let any Input number be represented by $n$.
The input increases by 1 each time.
The output increases by 3 each time.
This means that the expression for the output contains $3 n$.

Substitute several values of $n$ in $3 n$, then

|  | Input | Output |  |
| :---: | :---: | :---: | :---: |
|  | 1 | 2 | +3 |
| +1 | 2 | 5 | + |
| +1 | 3 | 8 | +3 |
| +1 | 4 | 11 | +3 |
| +1 | 5 | 14 | +3 | look for a pattern.

When $n=1,3 n=3(1)=3$
When $n=2,3 n=3(2)=6$
When $n=3,3 n=3(3)=9$
When $n=4,3 n=3(4)=12$

Each value is 1 more than the output above.
That is, the output is
1 less than each value.

So, the output is $3 n-1$.
The table shows how $3 n-1$ relates to $n$.

## Another Solution

Another way to solve this problem is to notice that each output is 1 less than a multiple of 3 .
So, the output is $3 \times n-1$, or $3 n-1$.
The table shows how $3 n-1$ relates to $n$.

| Input | Output |
| :---: | :---: |
| 1 | $2=3 \times 1-1$ |
| 2 | $5=3 \times 2-1$ |
| 3 | $8=3 \times 3-1$ |
| 4 | $11=3 \times 4-1$ |
| 5 | $14=3 \times 5-1$ |
| $n$ | $3 \times n-1$ |

## Practice

1. Copy and complete each table.

Explain how the Output number is related to the Input number.
a)

| Input | Output <br> $x$ |
| :---: | :---: |
| 1 | $2 x$ |
| 2 |  |
| 3 |  |
| 4 |  |
| 5 |  |

b)

| Input | Output |
| :---: | :---: |
| $m$ | $10-m$ |
| 1 |  |
| 2 |  |
| 3 |  |
| 4 |  |
| 5 |  |

c)

| Input | Output |
| :---: | :---: |
| $p$ | $3 p+5$ |
| 1 |  |
| 2 |  |
| 3 |  |
| 4 |  |
| 5 |  |

2. Use algebra. Write a relation for each Input/Output table.
a)

| Input <br> $n$ | Output |
| :---: | :---: |
| 1 | 7 |
| 2 | 14 |
| 3 | 21 |
| 4 | 28 |

b)

| Input <br> $n$ | Output |
| :---: | :---: |
| 1 | 4 |
| 2 | 7 |
| 3 | 10 |
| 4 | 13 |

c)

| Input <br> $n$ | Output |
| :---: | :---: |
| 1 | 1 |
| 2 | 3 |
| 3 | 5 |
| 4 | 7 |

3. Assessment Focus For each table, find the output. Explain how the numbers 3 and 4 in each relation affect the output.
a)

| Input | Output |
| :---: | :---: |
| $n$ | $3 n+4$ |
| 1 |  |
| 2 |  |
| 3 |  |
| 4 |  |

b)

| Input | Output |
| :---: | :---: |
| $n$ | $4 n+3$ |
| 1 |  |
| 2 |  |
| 3 |  |
| 4 |  |

4. Use algebra. Write a relation for each Input/Output table.
a)

| Input <br> $X$ | Output |
| :---: | :---: |
| 1 | 5 |
| 2 | 8 |
| 3 | 11 |
| 4 | 14 |

b)

| Input <br> $x$ | Output |
| :---: | :---: |
| 1 | 1 |
| 2 | 7 |
| 3 | 13 |
| 4 | 19 |

c)

| Input <br> $\boldsymbol{x}$ | Output |
| :---: | :---: |
| 1 | 8 |
| 2 | 13 |
| 3 | 18 |
| 4 | 23 |

## 5. Take It Further

a) Describe the patterns in this table.
b) Use the patterns to extend the table 3 more rows.
c) Use algebra.

Write a relation that describes how the output is related to the input.

| Input <br> $x$ | Output |
| :---: | :---: |
| 5 | 1 |
| 15 | 3 |
| 25 | 5 |
| 35 | 7 |
| 45 | 9 |
| 55 | 11 |

## Reflect

Your friend missed today's lesson. Explain how to write the relation represented by an Input/Output table.

## Mid-Unit Review

## LESSON

1. Which numbers are divisible by 4 ? By 8? How do you know?
a) 932
b) 1418
c) 5056
d) 12160
e) 14436
2. Draw a Venn diagram with 2 loops. Label the loops: "Divisible by 3 " and "Divisible by 5." Sort these numbers: 5485123735174037566195 What is true about the numbers in the overlapping region?
3. Use the divisibility rules.

Find the factors of each number.
a) 85
b) 136
c) 270
4. Write an algebraic expression for each statement.
Let $n$ represent the number.
a) seven more than a number
b) a number multiplied by eleven
c) a number divided by six
d) three less than four times a number
e) the sum of two and five times a number
5. Predict which expression in each pair will have the greater value when $y$ is replaced with 8. Evaluate to check your predictions.
a) i) $y+7$
ii) $2 y$
b) i) $6 y$
ii) $9-y$
c) i) $\frac{y+4}{2}$
ii) $\frac{y}{2}+4$
d) i) $2 y+6$
ii) $3 y-6$
6. i) For each number pattern, how is each term related to the term number?
ii) Let $n$ represent the term number. Write a relation for the term.
a)

b)

7. Dave pays to practise in a music studio. He pays $\$ 12$ each month, plus $\$ 2$ for each hour he practises.
a) Write a relation for the total cost for one month, in dollars, when Dave practises $t$ hours.
b) How much will Dave pay to practise 10 h in one month? 20 h ?
c) How does the relation change when the cost per hour doubles?
8. Use algebra. Write a relation for each Input/Output table.
a)

| Input <br> $x$ | Output |
| :---: | :---: |
| 1 | 7 |
| 2 | 11 |
| 3 | 15 |
| 4 | 19 |

b) | Input |  |
| :---: | :---: |
| $x$ | Output |
| 1 | 5 |
| 2 | 13 |
| 3 | 21 |
| 4 | 29 |

## 1.6 Graphing Relations

Focus Create a table of values, then graph.

We can use a graph to show the relationship between two quantities.
What does this graph show?
How many jellybeans are in each bag?
Write a relation for the total number of jellybeans in $n$ bags.

Jellybeans in Bags


## Explore

You will need grid paper.

The cost of $n$ CDs, in dollars, is $12 n$.

- What is the cost of one CD?
- Copy and complete this table.
> Graph the data.
Use the graph to answer these questions:
> What is the cost of 5 CDs?
- How many CDs could you buy with $\$ 72$ ?
\(\left.$$
\begin{array}{|c|c|}\hline \begin{array}{c}\text { Number of } \\
\text { CDs }\end{array} & \begin{array}{c}\text { Cost } \\
(\$) \\
n\end{array}
$$ <br>

\hline 0 \& 12 n\end{array}\right]\)| 2 |
| :--- |
| 4 |
| 6 |
| 8 |



## Reflect \& Share

Describe the patterns in the table. How are these patterns shown in the graph? If you had $\$ 50$, how many CDs could you buy?

## Connect

This table shows how $4 n+2$ relates to $n$, where $n$ is a whole number.
We could have chosen any Input numbers, but to see patterns it helps to use consecutive numbers.

These data are plotted on a graph.
The input is plotted on the horizontal axis.
The output is plotted on the vertical axis.


On the vertical axis, the scale is 1 square for every 2 units.
The graph also shows how $4 n+2$ relates to $n$.
When we place a ruler along the points, we see the graph is a set of points that lie on a straight line. When points lie on a straight line, we say the relation is a linear relation.
Since no numbers lie between the Input values in the table, it is not meaningful to join the points with a solid line.

The graph shows that each time the input increases by 1, the output increases by 4 .


## Example

Mr. Beach has 25 granola bars.
He gives 3 granola bars to each student who stays after school to help prepare for the school concert.
a) Write a relation to show how the number of granola bars that remain is related to the number of helpers.
b) Make a table to show this relation.
c) Graph the data. Describe the graph.
d) Use the graph to answer these questions:
i) How many granola bars remain when 7 students help?
ii) When will Mr. Beach not have enough granola bars?


## A Solution

a) Let $n$ represent the number of helpers. Each helper is given 3 granola bars. So, the number of granola bars given to $n$ helpers is $3 n$.
There are 25 granola bars.
The number of granola bars that remain is $25-3 n$.
So, $n$ is related to $25-3 n$.
c) On the vertical axis, use a scale
of 1 square for every 2 units.
b) Substitute each value of $n$ into $25-3 n$.

| Number of <br> Helpers <br> $\boldsymbol{n}$ | Number of <br> Granola Bars Left <br> $25-3 \boldsymbol{n}$ |
| :---: | :---: |
| 0 | $25-3(0)=25$ |
| 1 | $25-3(1)=22$ |
| 2 | $25-3(2)=19$ |
| 3 | $25-3(3)=16$ |
| 4 | $25-3(4)=13$ |
| 5 | $25-3(5)=10$ |

The points lie on a line so the graph represents a linear relation.
When the input increases by 1 , the output decreases by 3 .

The graph goes down to the right.
This is because the number of granola bars that remain decreases as the number of helpers increases.
d) i) To find the number of granola bars that remain, extend the graph. The points lie on a straight line. Extend the graph to 7 helpers. There are 4 granola bars left.
ii) Continue to extend the graph. 25 granola bars are enough for 8 helpers, but not for 9 helpers. Mr . Beach will not have enough granola bars for 9 or more helpers.


## Practice

1. Copy and complete this Input/Output table for each relation.
a) $4 n$ is related to $n$.
b) $x+3$ is related to $x$.
c) $4 c+6$ is related to $c$.
2. Graph each relation in question 1.

Suggest a real-life situation it could represent.
3. a) Copy and complete this Input/Output table to show how $6 a-4$ is related to $a$.
b) Graph the relation.

What scale did you use on the vertical axis? How did you make your choice.
c) Explain how the graph illustrates the relation.
4. Look at the graph on the right.
a) What is the output when the input is 1 ?
b) Which input gives the output 18 ?
c) Extend the graph. What is the output when the input is 8 ?
d) Suggest a real-life situation this graph could represent.
5. Admission to Fun Place is $\$ 5$. Each go-cart ride costs an additional \$3.
a) Write a relation to show how the total cost is related to the number of go-cart rides.
b) Copy and complete this table.
c) Draw a graph to show the relation.

Describe the graph.
d) Use the graph to answer these questions:
i) Erik goes on 6 go-cart rides.

What is his total cost?

| Number of <br> Go-Cart <br> Rides | Total <br> Cost <br> (\$) |
| :---: | :---: |
| 0 |  |
| 1 |  |
| 2 |  |
| 3 |  |
| 4 |  |
| 5 |  |

ii) Before entering the park, Lydia has $\$ 30$. How many go-cart rides can she afford?
6. Match each graph to its relation.
a) The number of seashells collected is related to the number of students who collected.
There are 12 seashells to start.
Each student collects 3 seashells.
b) The number of counters on the teacher's desk is related to the number of students who remove counters.
There are 36 counters to start.
Each student removes 6 counters.
c) The money earned baby-sitting is related to the number
 of hours worked. The baby-sitter earns $\$ 6 / \mathrm{h}$.
i)

ii)

iii)

7. Akuti borrows $\$ 75$ from her mother to buy a new lacrosse stick. She promises to pay her mother $\$ 5$ each week until her debt is paid off.
a) Write a relation to show how the amount Akuti owes is related to the number of weeks.
b) Make a table for the amount owing after 2, 4, 6, 8, and 10 weeks.
c) Draw a graph to show the relation. Describe the graph.
d) Use the graph to answer these questions:
i) How much does Akuti owe her mother after 13 weeks?
ii) When will Akuti finish paying off her debt?
8. Assessment Focus Use the relation: $5 n+6$ is related to $n$
a) Describe a real-life situation that could be represented by this relation.
b) Make a table of values using appropriate Input numbers.
c) Graph the relation. Describe the graph.
d) Write 2 questions you could answer using the graph.

Answer the questions.

## Reflect

How can the graph of a relation help you answer questions about the relation? Use an example to show your thinking.

