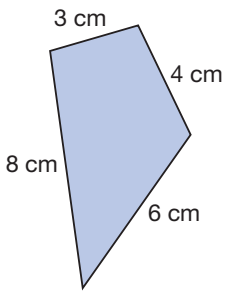
LESSON 7

Perimeters of Polygons

What is the perimeter of this quadrilateral?



Explore

You will need geoboards, geobands, dot paper, and rulers.
Share the work.

Make 15 different polygons.
Make sure there are at least two of each of these types of polygons:

- square
- rectangle
- parallelogram
- rhombus
- triangle

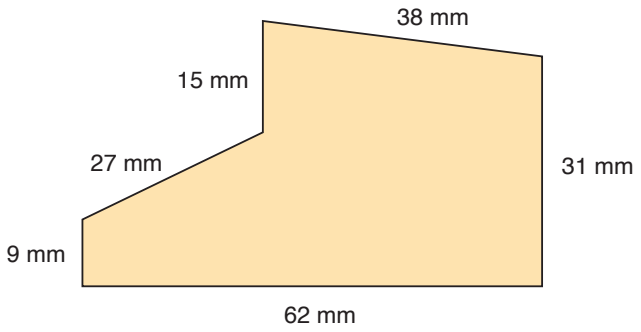
Record each polygon on dot paper.
Find the perimeter of each polygon.
For which types of polygons can you write a rule to calculate the perimeter? Write these rules.

Show *and* Share

Share your rules with another group of students.
Compare your rules. Discuss any differences.
For which types of polygons is it possible to write more than one rule? Explain.

Connect

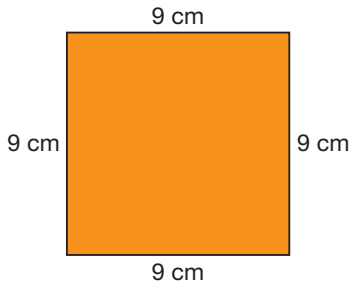
Perimeter is the distance around a polygon.
You discovered that we can use rules to find the perimeter of polygons.
For this hexagon:



Perimeter $= 38 + 31 + 62 + 9 + 27 + 15$
$= 182$
The perimeter of this hexagon is 182 mm.

Our rule is, for any polygon, we can find the perimeter by adding the side lengths.

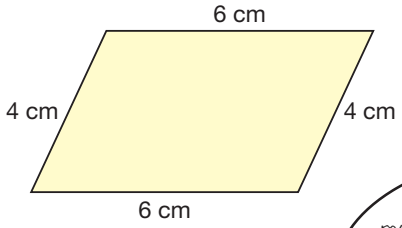
We can also develop rules that apply to specific polygons.
➤ Here is Katy's way to find the perimeter of this square.



Perimeter $= 9 + 9 + 9 + 9$
$= 4 \times 9$
$= 36$
The perimeter of this square is 36 cm.

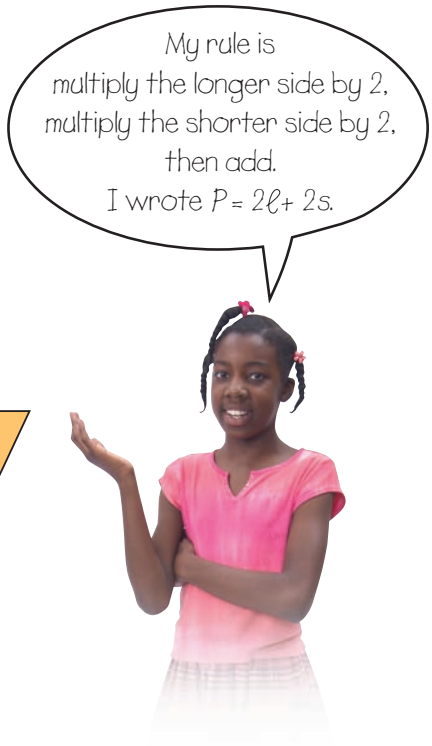
A square has 4 equal sides.
Katy says this suggests a rule for finding the perimeter of any square:
Multiply the side length by 4.

➤ Here is Graeme's way to find the perimeter of this parallelogram.

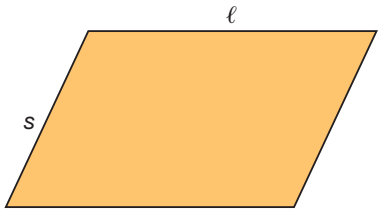


Perimeter $= 6 + 4 + 6 + 4$
$= (6 + 4) + (6 + 4)$
$= 2 \times (6 + 4)$
$= 2 \times 10$
$= 20$

The perimeter of this parallelogram is 20 m.

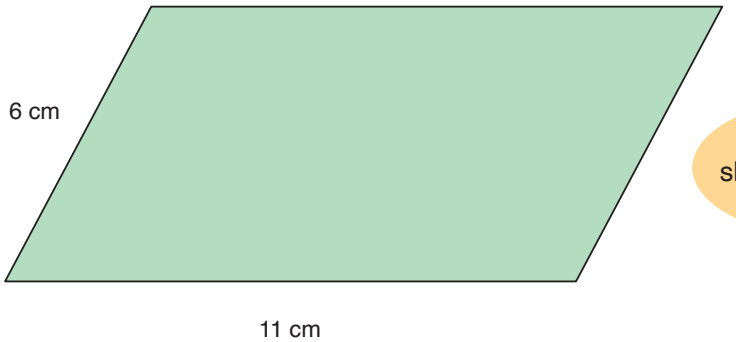


A parallelogram has two pairs of congruent sides.
Graeme says this suggests a rule for finding the perimeter of any parallelogram:
Add the measures of a longer side and a shorter side, then multiply by 2.



A rule for finding the perimeter of any parallelogram is:
Perimeter $= 2 \times (\ell + s)$

➤ We can use these **formulas** to find the perimeter of the parallelogram below.



A *formula* is a short way to state a rule.

$P = 2 \times (\ell + s)$ $\qquad$ $P = 2\ell + 2s$

We replace each variable ℓ and s with the given side lengths.

When we replace a variable with a number, we *substitute*.

$P = 2 \times (11 + 6)$
$= 2 \times 17$
$= 34$

$P = 2(11) + 2(6)$
$= 22 + 12$
$= 34$

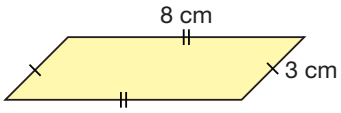
The perimeter of this parallelogram is 34 cm.

We can check by adding the lengths of the 4 sides:
11 cm + 6 cm + 11 cm + 6 cm = 34 cm
This is the same as the answers we got using the formulas.

Practice

1. Find the perimeter of each polygon.

a)



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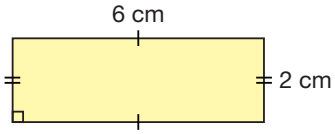
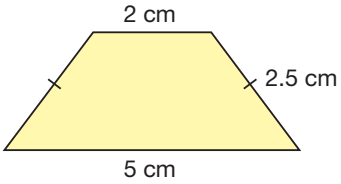


c)



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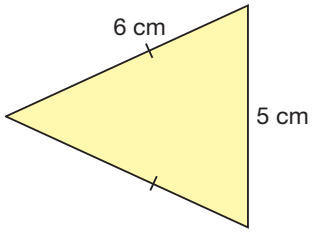
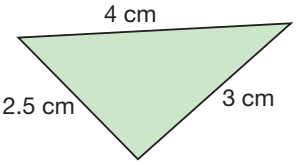


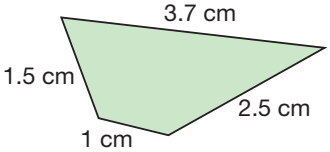
2. Describe the strategy you used to find the perimeter of each polygon in question 1.

3. Find the perimeter of each polygon.

a)

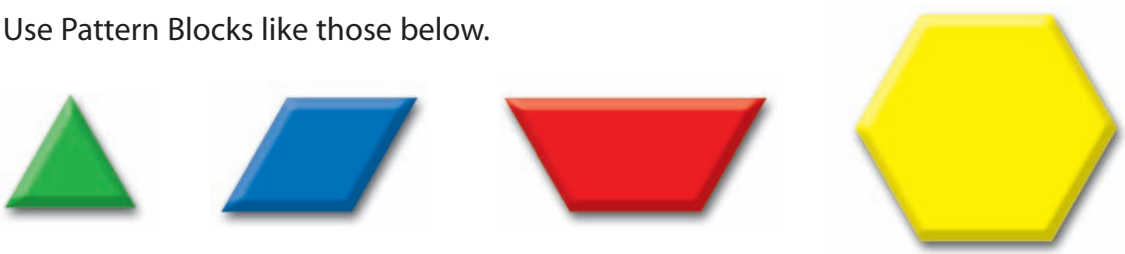


b)



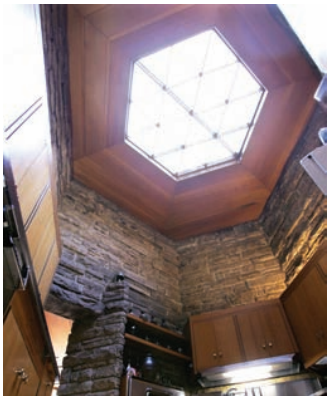
Can you write a rule to find the perimeter of each of these polygons? Why or why not?

4. Use Pattern Blocks like those below.

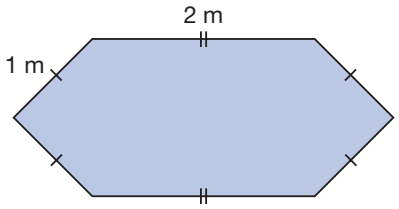


Write a rule to find the perimeter of each Pattern Block.

5. Aldo wants to install a skylight in the roof of his house. The base of the skylight is a regular hexagon with side length 40 cm. What is the perimeter of the base of the skylight? Give your answer in metres.
Which strategy did you use to find out?

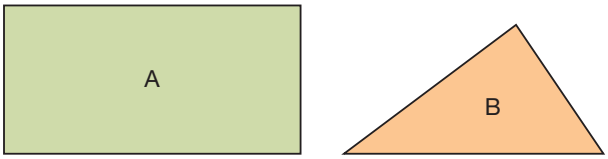


6. Winnie is building a hexagonal storage box.
Here is a drawing of the top of the box.



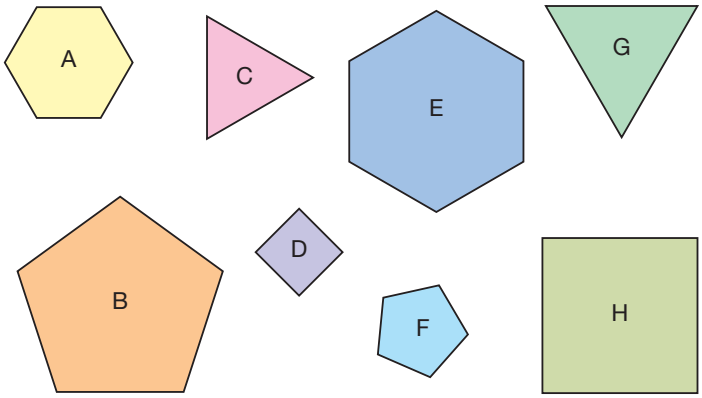
a) Write a rule to find the perimeter of the top of the box.
b) Write the rule as a formula.
c) What is the perimeter of the top of the box?

7. a) Find the perimeter of each polygon.



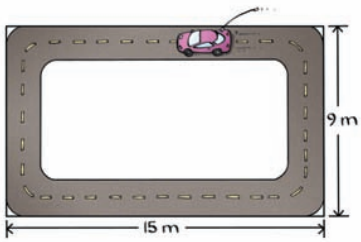
b) Suppose the side lengths of each polygon are doubled.
What would happen to each perimeter? Explain.

8. Your teacher will give you a large copy of these regular polygons.



a) Find and record the perimeter of each polygon.
b) How is the perimeter of a regular polygon related to the number of its sides?
Write a formula to find the perimeter of a regular polygon.

9. Saki has a remote control car. She enters her car in a race.
The track is close to rectangular.



a) Use a formula to find the perimeter of the track.
b) Suppose the car completes 8 laps.
How far did the car travel?

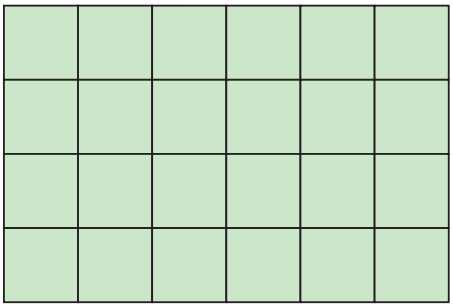
Reflect

How are the side lengths of a polygon and its perimeter related?
Use examples to explain.

LESSON 8

Area of a Rectangle

What is the area of this rectangle?
How did you find out?



Explore

You will need 1-cm grid paper.

- Draw a 2-cm by 3-cm rectangle.
  Find the area of the rectangle.
- Suppose the length of the rectangle doubles.
  Predict the area of the new rectangle.
  Check your prediction.
- Suppose the width of the original rectangle doubles. Predict the area of the new rectangle.
  Check your prediction.
- Suppose both the length and the width double.
  Predict the area of the new rectangle.
  Check your prediction.
- How does the area of each new rectangle compare to the area of the original rectangle?
- Write a rule to calculate the area of a rectangle.
  Write the rule as a formula.
  Use the formula to check the area of the rectangles you drew.

Show *and* Share

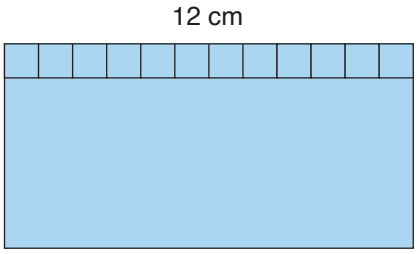
Share your work with another pair of students.
Compare your formulas.
What do you think happens to the area of a rectangle when the length triples? The width triples? Both the length and the width triple?
How could you use your formula to find out?

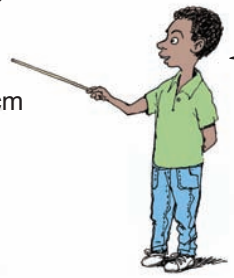
Lesson Focus | Develop and apply a formula to determine the area of a rectangle.

Connect

We can find a shortcut for calculating the area of a rectangle.

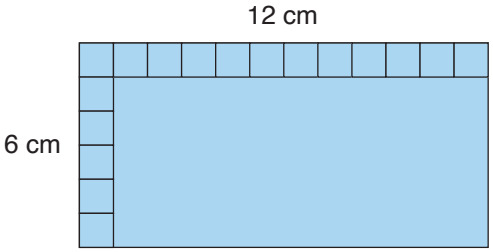
Measure the length of the rectangle.





The length tells how many 1-cm squares fit along it. The length is 12 cm. So, twelve 1-cm squares fit along the length.

Measure the width of the rectangle.

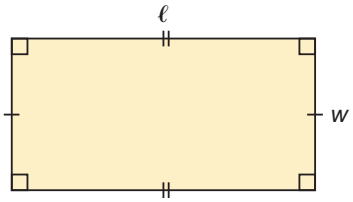


The width tells how many rows of 1-cm squares fit in the rectangle. The width is 6 cm, so there are 6 rows.

Multiply the length by the width.
$12 \times 6 = 72$
So, the area of the rectangle is 72 cm^2.

To find how many 1-cm squares fit in the rectangle, we multiply the length of a row by the number of rows.

We can write this rule:
To find the area of a rectangle, multiply the length by the width.

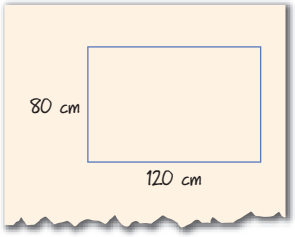


This rule can be expressed as a formula.

Area = length × width
$A = \ell \times w$

We use: A to represent area, ℓ to represent length, and w to represent width.

Edmond built a dog crate for his dog.
The floor of the crate is a rectangle.
The dimensions of the floor are 80 cm by 120 cm.



➤ You can use the formula for the area of a rectangle to find the floor area of the crate.

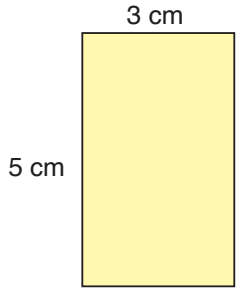
$A = \ell \times w$
$= 120 \times 80$
$= 9600$

The floor area of the crate is 9600 cm^2.

Practice

1. Find the area of each rectangle.

a)



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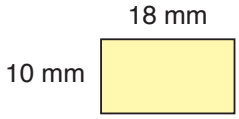
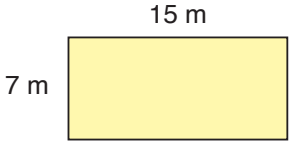
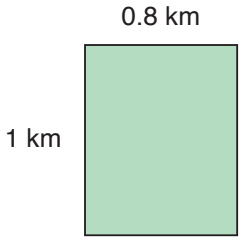


c)



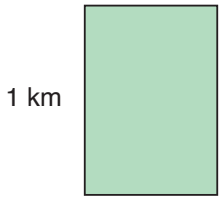
2. Which rectangle below do you think has the greatest area?
Estimate first. Use a formula to check.
Order the areas from least to greatest.
How does the order compare with your prediction?

a)



b)

c)



3. Copy and complete this chart.

Rectangle	Length (cm)	Width (cm)	Area (cm^2)
A	7	5	?
B	?	6	12.6
C	3	?	13.5
D	5.3	7	?

Which strategy did you use to find the missing number each time?

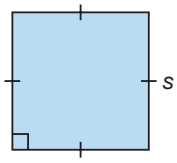
4. Matt's dog has a rectangular dog run.
The length of the dog run is 8 m. The total area enclosed is 56 m^2.
How wide is the dog run? Draw a diagram.
How can you use a number sentence to show your thinking?

5. Lena used 36 m of fencing to enclose a rectangular vegetable garden on her farm in Battleford, Saskatchewan.
   a) Sketch some possible rectangles and label their side lengths. What is the area of the enclosed section in each case?
   b) How many different answers can you find?

6. A banner for the Vancouver 2010 Olympics has length 226 cm and width 72 cm. What is the area of the banner?

7. Hailey bought a can of stain. The stain will cover 50 m^2 of fencing. The fence has height 2 m.
What length of fencing can Hailey stain before she runs out of stain? How did you find out?

8. A square has side length *s*.



Write a formula for the area of a square.

9. The Festival du Voyageur is a winter festival that takes place in St. Boniface, Manitoba, each February.
The festival's logo contains a red rectangle.
Suppose the logo is enlarged so the rectangle has width 4 cm and area 28.8 cm^2.
What is the length of the rectangle?
How did you find out?

10. Rectangle A has area 40 cm^2 and length 8 cm.
The area of Rectangle B is one-half the area of Rectangle A.
The rectangles have the same length.
What is the width of Rectangle B?

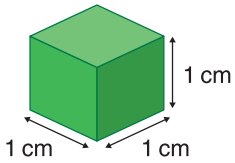
Reflect

When might you use the formula for the area of a rectangle outside the classroom?

LESSON 9

Volume of a Rectangular Prism

A centimetre cube has a length, width, and height of 1 cm.
What is its volume?



Explore

You will need 2 empty boxes and centimetre cubes.

- Choose one box.
  Estimate how many centimetre cubes the box can hold.
- Fill the bottom of the box with one layer of cubes.
  How many cubes are in that layer?
  How many layers can fit in the box?
  How do you know?
- How many cubes can the box hold altogether?
  Describe how you found your answer.
  Record your answer on the box.
- Without filling it completely, find how many cubes the second box can hold.
  Describe the strategy you used.
  Use cubes to check your answer.

Show *and* Share

Share the boxes you used with the class.
How can you find the volume of a box without filling it completely?
Will your answer be exact? Explain.
How can you find the volume of a box without using cubes?

Connect

A rectangular prism is 10 cm long, 5 cm wide, and 6 cm high.

The length is 10 cm.
It is 1 row of 10 cubes.
Volume of 1 row $= 10\ cm^3$

The width is 5 cm.
Five rows of 10 cubes make 1 layer of 50 cubes.
Volume of 1 layer $= 5 \times 10\ cm^3$
$= 50\ cm^3$

The height is 6 cm.
Six layers of 50 cubes make a volume of 300 cubes.
Volume of 6 layers $= 6 \times 50\ cm^3$
$= 300\ cm^3$

We can use the descriptions above to develop a formula for the volume of a rectangular prism.

Volume in cubic centimetres
= number of 1-cm cubes in each layer × number of layers

The number of cubes in each layer is the area of the base of the prism. It is the length times the width.

The number of layers is the height of the prism.

So, Volume = base area × height

Another way to write the formula is:

Volume = length × width × height

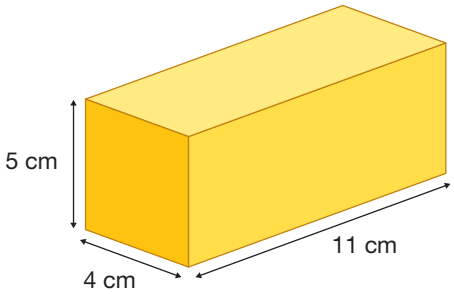
$V = \ell \times w \times h$

We use: V to represent volume, ℓ to represent length, w to represent width, and h to represent height.

➤ We can use the formula to find the volume of a rectangular prism 11 cm long, 4 cm wide, and 5 cm high.

$$\begin{aligned} \text{Volume} &= \ell \times w \times h \\ &= 11\ cm \times 4\ cm \times 5\ cm \\ &= 44\ cm^2 \times 5\ cm \\ &= 220\ cm^3 \end{aligned}$$

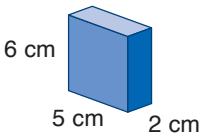
The volume of the prism is $220\ cm^3$.



Practice

1. Find the volume of each rectangular prism.

a)



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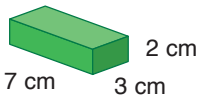

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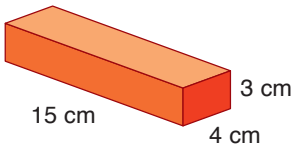


2. Estimate, then calculate, the volume of a rectangular prism with these dimensions.

	Length (cm)	Width (cm)	Height (cm)
a)	6	2	2
b)	9	4	7
c)	18	9	12
d)	30	15	6

3. A dog box is built to fit in the back of a pick-up truck. It is used to transport sled dogs and supplies to a race. A dog box that holds 3 dogs is 117 cm long, 97 cm wide, and 61 cm tall. Each dog compartment is 38 cm long, 97 cm wide, and 46 cm tall.

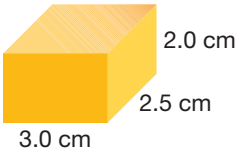
   a) What is the volume of each dog compartment?
   b) What is the volume of the dog box that is not used to hold dogs? How did you find out?

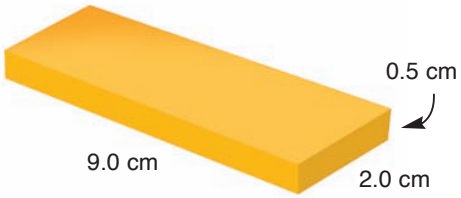
4. During the buffalo hunt, the Métis used a Red River cart to carry buffalo meat and fur. The cart was made of wood and was usually pulled by oxen. The top of this cart has the shape of a rectangular prism with volume 1 350 000 cm^3. The area of its base is about 13 500 cm^2. About how high is the top of the cart? Which strategy did you use to find out?

5. A rectangular prism has volume 90 cm^3.
The prism has length 9 cm and width 5 cm.
What is its height? How do you know?

6. A rectangular prism has volume 192 cm^3.
   a) The prism is 16 cm high. What is the area of its base? How do you know?
   b) What other possible measurements of height and base area could the rectangular prism have? What strategy did you use to find out?

7. Canada's Food Guide recommends that we eat 2 to 4 servings of dairy products every day.
   a) This piece of cheese is 1 serving of dairy products. What is its volume?





   b) Is the block of cheese at the right more or less than 1 serving? How do you know?

8. Each block in a child's set of building blocks is 15 cm long, 10 cm wide, and 5 cm high.
Suppose you put the blocks in a box that is 50 cm long, 35 cm wide, and 30 cm high.

   a) What is the volume of each block? Of the box?
   b) Suppose you only consider the volume. How many blocks would you expect to fit in the box?
   c) Suppose you arrange the blocks neatly in layers. How many different ways can you layer the blocks? How many blocks fit in the box each way?
   d) Compare your answers to parts b and c. Explain any differences.
   e) Which is the best way to pack the blocks? Why?

Reflect

Explain why the volume of a rectangular prism is the product of its length, width, and height. Include a diagram in your explanation.