UNIT 1

Patterns and Relations

Students in a Grade 7 class were raising money for charity.
Some students had a "bowl-a-thon."

This table shows the money that one student raised for different bowling times.

Time (h)	Money Raised ($)
1	8
2	16
3	24
4	32
5	40
6	48

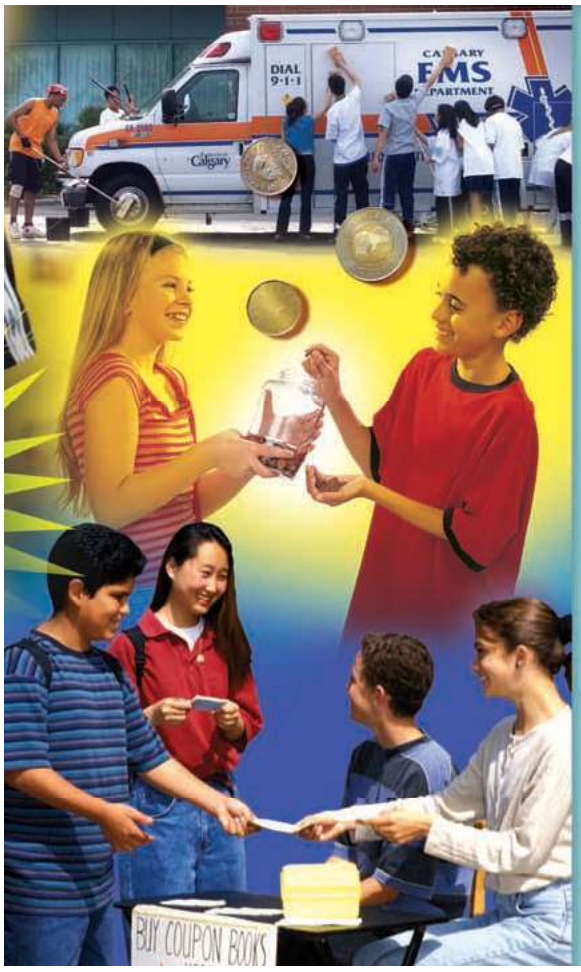
- What patterns do you see in the table?
- Extend the table. For how long would the student have to bowl to raise $72?

What You'll Learn

- Use patterns to explore divisibility rules.
- Translate between patterns and equivalent linear relations.
- Evaluate algebraic expressions by substitution.
- Represent linear relations in tables and graphs.
- Solve simple equations, then verify the solutions.

Why It's Important

- Divisibility rules help us find the factors of a number.
- Graphs provide information and are a useful problem-solving tool.
- Efficient ways to represent a pattern can help us describe and solve problems.



Key Words

- divisibility rules
- algebraic expression
- numerical coefficient
- constant term
- relation
- linear relation
- unit tile
- variable tile
- algebra tiles

1.1 Patterns in Division

Focus Explore divisibility by 2, 4, 5, 8, and 10.

Which of these numbers are divisible by 2? By 5? By 10?
How do you know?

- 78
- 27
- 35
- 410
- 123
- 2100
- 4126
- 795

Explore

You will need a hundred chart numbered 301–400, and three different coloured markers.

➤ Use a marker. Circle all numbers on the hundred chart that are divisible by 2.
Use a different marker.
Circle all numbers that are divisible by 4.
Use a different marker.
Circle all numbers that are divisible by 8.
Describe the patterns you see in the numbers you circled.

➤ Choose 3 numbers greater than 400.
Which of your numbers do you think are divisible by 2? By 4? By 8?
Why do you think so?

Reflect & Share

Share your work with another pair of classmates.
Suppose a number is divisible by 8.
What else can you say about the number?
Suppose a number is divisible by 4.
What else can you say about the number?

Connect

We know that 100 is divisible by 4: $100 \div 4 = 25$
So, any multiple of 100 is divisible by 4.
To find out if any whole number with 3 or more digits is divisible by 4, we only need to check the last 2 digits.

To find out if 352 is divisible by 4, check if 52 is divisible by 4.
$52 \div 4 = 13$
52 is divisible by 4, so 352 is divisible by 4.

To check if a number, such as 1192, is divisible by 8,
think: $1192 = 1000 + 192$
We know 1000 is divisible by 8: $1000 \div 8 = 125$
So, we only need to check if 192 is divisible by 8.
Use mental math. $192 \div 8 = 24$
192 is divisible by 8, so 1192 is divisible by 8.

Another way to check if a number is divisible by 8 is to divide by 4. If the quotient is even, then the number is divisible by 8.

All multiples of 1000 are divisible by 8.
So, for any whole number with 4 or more digits, we only need to check the last 3 digits to find out if the number is divisible by 8.

A number that is divisible by 8 is also divisible by 2 and by 4 because $8 = 2 \times 4$.
So, a number divisible by 8 is even.

2 and 4 are factors of 8.

You can use patterns to find **divisibility rules** for other numbers.

1	2	3	4	5	6	7	8	9	10
11	12	13	14	15	16	17	18	19	20
21	22	23	24	25	26	27	28	29	30
31	32	33	34	35	36	37	38	39	40
41	42	43	44	45	46	47	48	49	50
51	52	53	54	55	56	57	58	59	60
61	62	63	64	65	66	67	68	69	70
71	72	73	74	75	76	77	78	79	80
81	82	83	84	85	86	87	88	89	90
91	92	93	94	95	96	97	98	99	100

➤ All multiples of 10, such as 30, 70, and 260, end in 0.

Any number whose ones digit is 0, is divisible by 10.

➤ Here are some multiples of 5.
5, 10, 15, 20, 25, 30, 35, 40, …, 150, 155, 160, …
The ones digits form a repeating pattern.
The core of the pattern is: 5, 0

Any number whose ones digit is 0 or 5, is divisible by 5.

Every multiple of 5 has a ones digit of 0 or 5.

➤ Multiples of 2 are even numbers: 2, 4, 6, 8, 10, …
All even numbers are divisible by 2.

Any number whose ones digit is even, is divisible by 2.

Example

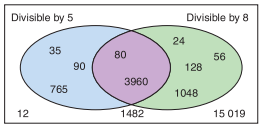
Which numbers are divisible by 5? By 8? Both by 5 and by 8?
How do you know?
12, 24, 35, 56, 80, 90, 128, 765, 1048, 1482, 3960, 15 019

A Solution

Any number with 0 or 5 in the ones place is divisible by 5.
So, the numbers divisible by 5 are: 35, 80, 90, 765, 3960

The divisibility rule for 8 only applies when a number is 1000 or greater.
For numbers less than 1000, use mental math or a calculator.
All multiples of 8 are even, so reject 35, 765, and 15 019.
Use mental math to identify that 12 and 90 are not divisible by 8.
Use mental math to identify that 24, 56, 80, and 128 are divisible by 8.
1048 and 3960 are divisible by 8 because 48 and 960 are divisible by 8.
1482 is not divisible by 8 because 482 is not divisible by 8.

We can display the results in a Venn diagram.



The numbers in the overlapping region are divisible both by 5 and by 8.
So, 80 and 3960 are also divisible by 40, since $5 \times 8 = 40$.

Practice

1. Which numbers are divisible by 2? By 5?
How do you know?
a) 106 **b)** 465 **c)** 2198
d) 215 **e)** 1399 **f)** 4530

2. Explain why a number with 0 in the ones place is divisible by 10.

3. Which numbers are divisible by 4? By 8? By 10?
How do you know?
a) 212 **b)** 512 **c)** 5450
d) 380 **e)** 2132 **f)** 12 256

4. Maxine and Tony discuss divisibility.
Maxine says, "260 is divisible by 4 and by 5.
$4 \times 5 = 20$, so 260 is also divisible by 20."
Tony says, "148 is divisible by 2 and by 4.
$2 \times 4 = 8$, so 148 is also divisible by 8."
Are both Maxine and Tony correct? Explain your thinking.

5. Write 3 numbers that are divisible by 8.
How did you choose the numbers?

6. Assessment Focus
a) Use the divisibility rules for 2, 4, and 8 to sort these numbers.

1046	322	460	1784	28
54	1088	224	382	3662

b) Draw a Venn diagram with 3 loops.
Label the loops: "Divisible by 2," "Divisible by 4," and "Divisible by 8"
Explain why you drew the loops the way you did.
Place the numbers in part a in the Venn diagram.
How did you decide where to place each number?
c) Find and insert 3 more 4-digit numbers in the Venn diagram.

7. Use the digits 0 to 9. Replace the □ in each number to make a number divisible by 4. Find as many answers as you can.
a) 822□ **b)** 211 4□8 **c)** 15 □32

8. Take It Further A leap year occurs every 4 years.
The years 1992 and 2004 were leap years.
What do you notice about these numbers?
Was 1964 a leap year? 1852? 1788? Explain.

Reflect

Compare the divisibility rules for 4 and 8.
How can you use one rule to help you remember the other?

1.2 More Patterns in Division

Focus Explore divisibility by 0, 3, 6, and 9.

Division can be thought of as making equal groups.

For $20 \div 4$, we make 4 equal groups of 5.

Explore

Use a calculator.

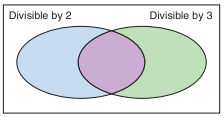
➤ Choose 10 different numbers.
Divide each number by 0.
What do you notice?
What do you think this means?

➤ Choose 15 consecutive 2-digit numbers.
Divide each number by 3 and by 9.
Repeat for 15 consecutive 3-digit numbers.

List the numbers that were divisible by 3 and by 9.
Find the sum of the digits of each number.
What do you notice?

Choose 4 different 4-digit numbers you think are divisible by 3 and by 9.
Divide each number by 3 and by 9 to check.
Add the digits in each number. What do you notice?

➤ Draw this Venn diagram.
Sort these numbers.
12 21 42 56 88 135 246 453 728
What can you say about the numbers in the overlapping region?



Reflect & Share

Share your work with another pair of classmates.
Explain how to choose a 4-digit number that is divisible by 3.
Without dividing, how can you tell if a number is divisible by 6? By 9?
Why do you think a number cannot be divided by 0?

Connect

We can use divisibility rules to find the factors of a number, such as 100.
Any number is divisible by 1 and itself,
so 1 and 100 are factors of 100.
100 is even, so 100 is divisible by 2.
We know 100 is divisible by 4.
The ones digit is 0, so 100 is divisible by 5 and by 10.
100 is not divisible by 3, 6, 8, or 9.
The factors of 100, from least to greatest, are:
1, 2, 4, 5, 10, 20, 25, 50, 100

$100 \div 1 = 100$
$100 \div 2 = 50$
$100 \div 4 = 25$
$100 \div 5 = 20$
$100 \div 10 = 10$

Factors occur in pairs.
When we find one factor of a number, we also find a second factor.

A whole number cannot be divided by 0.
We cannot take a given number and share it into zero equal groups.
We cannot make sets of zero from a given number of items.

Example

Edward has 16 souvenir miniature hockey sticks.
He wants to share them equally among his cousins.
How many sticks will each cousin get if Edward has:
a) 8 cousins? **b)** 0 cousins?
Explain your answer to part b.

A Solution

a) There are 16 sticks. Edward has 8 cousins.
$16 \div 8 = 2$
Each cousin will get 2 sticks.
b) There are 16 sticks. Edward has no cousins.
16 sticks cannot be shared equally among no cousins.
This answer means that we cannot divide a number by zero.
We cannot divide 16 by 0 because 16 cannot be shared into zero equal groups.

You have sorted numbers in a Venn diagram. You can also use a *Carroll diagram* to sort numbers.
Here is an example:

	Divisible by 3	Not Divisible by 3
Divisible by 8	24, 120, 1104, 12 096	32, 224, 2360
Not Divisible by 8	12, 252, 819, 11 337	10, 139, 9212

Divisibility Rules

A whole number is divisible by:

2 if the number is even

3 if the sum of the digits is divisible by 3

4 if the number represented by the last 2 digits is divisible by 4

5 if the ones digit is 0 or 5

6 if the number is divisible by 2 and by 3

8 if the number represented by the last 3 digits is divisible by 8

9 if the sum of the digits is divisible by 9

10 if the ones digit is 0

Practice

1. Which numbers are divisible by 3? By 9? How do you know?
a) 117 **b)** 216 **c)** 4125 **d)** 726 **e)** 8217 **f)** 12 024

2. Write 3 numbers that are divisible by 6. How did you choose the numbers?

3. Which of these numbers is 229 344 divisible by? How do you know?
a) 2 **b)** 3 **c)** 4 **d)** 5 **e)** 6 **f)** 8 **g)** 9 **h)** 10

4. Use the divisibility rules to find the factors of each number.
How do you know you have found all the factors?
a) 150 **b)** 95 **c)** 117 **d)** 80

5. Use a Carroll diagram.
Which numbers are divisible by 4? By 9? By 4 and by 9? By neither 4 or 9?
144 128 252 153 235 68 120 361 424 468

6. I am a 3-digit number that has a 2 in the hundreds place.
I am divisible by 3, 4, and 5. Which number am I?

7. **Assessment Focus**
a) Write a 3-digit number that is divisible by 5 and by 9.
How did you choose the number?
b) Find the factors of the number in part a. Use the divisibility rules to help you.
c) How would you find the greatest 3-digit number that is divisible by 5 and by 9? The least 3-digit number? Explain your methods.

8. Use the digits 0 to 9.
Replace the □ in each number to make a number divisible by 3.
Find as many answers as you can.
a) 4□6 **b)** 1□32 **c)** 24 71□

9. Suppose you have 24 cereal bars.
You must share the bars equally with everyone in the classroom.
How many cereal bars will each person get, in each case?
a) There are 12 people in the classroom.
b) There are 6 people in the classroom.
c) There is no one in the classroom.
d) Use your answer to part c.
Explain why a number cannot be divided by 0.

10. **Take It Further** Universal Product Codes (UPCs) are used to identify retail products.
The codes have 12 digits, and sometimes start with 0.
To check that a UPC is valid, follow these steps:
- Add the digits in the odd-numbered positions (1st, 3rd, 5th,…).
- Multiply this sum by 3.
- To this product, add the digits in the even-numbered positions.
- The result should be a number divisible by 10.

Look at this UPC. Is it a valid code? Explain.
Find 2 UPC labels on products at home.
Check to see if the codes are valid. Record your results.

Reflect

Which divisibility rules do you find easiest to use?
Which rules do you find most difficult? Justify your choices.