

2.1 Representing Integers

Focus Use coloured tiles to represent integers.

One of the coldest places on Earth is Antarctica, with an average annual temperature of about -58°C . This is a **negative integer**.



One of the hottest places on Earth is Ethiopia, with an average annual temperature of about $+34^{\circ}\text{C}$. This is a **positive integer**.



We can use yellow tiles to represent positive integers and red tiles to represent negative integers.

One yellow tile can represent $+1$.

One red tile can represent -1 .

A red tile and a yellow tile combine to model 0: $\left. \begin{array}{l} \text{red tile} \\ \text{yellow tile} \end{array} \right\} \begin{array}{l} -1 \\ +1 \end{array}$ We call this a **zero pair**.

Explore

You will need coloured tiles.

- ▶ One of you uses 9 tiles and one uses 10 tiles. You can use any combination of red and yellow tiles each time. How many different integers can you model with 9 tiles? How many different integers can your partner model with 10 tiles?
- ▶ Draw a picture to show the tiles you used for each integer you modelled. Circle the zero pairs. Write the integer each picture represents. How do you know?



Practice

1. Write the integer modelled by each set of tiles.

- a) b) c)
- d) e) f)

2. Draw yellow and red tiles to model each integer in two different ways.

- a) -6 b) $+7$ c) $+6$ d) -2
e) $+9$ f) -4 g) 0 h) $+10$

3. Work with a partner.

- Place 10 yellow and 10 red tiles in a bag.
a) Suppose you draw 6 tiles from the bag. What integers might the tiles model? List all seven possible integers.
b) Without looking, draw 6 tiles from the bag. Record the integer that these tiles model. Repeat the experiment 9 more times. Which integer was modelled most often?



Math Link

Sports

In golf, a hole is given a value called **par**. Par is the number of strokes a good golfer takes to reach the hole. A score of $+2$ means a golfer took 2 strokes more than par, or 2 strokes over par. A score of -1 means a golfer took 1 stroke fewer than par, or 1 stroke under par. Some scores have special names. A score of $+1$ is a bogey. A score of -1 is a birdie. A score of -2 is an eagle.

In a golf tournament, the golfer with the fewest strokes wins the game.

Reflect & Share

Compare your models with those of your partner. Which integers did you model? Your partner? Were you able to model any of the same integers? Why or why not?

Connect

We can model any integer in many ways.

Each set of tiles below models $+5$.

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-
-
-
-
-
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Each pair of 1 yellow tile and 1 red tile makes a zero pair. The pair models 0.

Example

Use coloured tiles to model -4 in three different ways.

A Solution

Start with 4 red tiles to model -4 . Add different numbers of zero pairs. Each set of tiles below models -4 .

-
-
-

Adding 4 zero pairs does not change the value.

Adding 2 zero pairs does not change the value.

Adding 7 zero pairs does not change the value.

4. Assessment Focus

- a) Choose an integer between -9 and $+6$. Use coloured tiles to model the integer.
 - b) How many more ways can you find to model the integer with tiles? Create a table to order your work.
 - c) What patterns can you find in your table?
 - d) Explain how the patterns in your table can help you model an integer between -90 and $+60$.
5. a) Suppose you have 10 yellow tiles, and use all of them. How many red tiles would you need to model $+2$? How do you know?
b) Suppose you have 100 yellow tiles, and use all of them. How many red tiles would you need to model $+2$? How do you know?
6. Write the integer suggested by each of the following situations. Draw yellow or red tiles to model each integer. Explain your choice.
 - a) You move your game piece forward 9 squares on the game board.
 - b) You ride down 5 floors on an elevator.
 - c) You walk up 11 stairs.
 - d) The temperature drops 9°C .
 - e) You climb down 7 rungs on a ladder.
7. Write two integers suggested by each of the following situations.
 - a) You deposit \$100 in your bank account, then pay back your friend \$20.
 - b) While shopping in a large department store, you ride the elevator up 6 floors, then down 4 floors.
 - c) The temperature rises 12°C during the day, then falls 8°C at night.



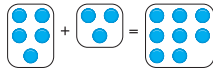
Reflect

How is it possible to use coloured tiles to model any integer in many different ways?

2.2 Adding Integers with Tiles

Focus Use coloured tiles to add integers.

Recall that when you add two numbers, such as $5 + 3$, you can show the addition by combining 5 counters with 3 counters to obtain 8 counters. You can add two integers in a similar way. You know that $+1$ and -1 combine to make a zero pair. We can combine coloured tiles to add integers.



Explore

You will need coloured tiles.

- Choose two different positive integers. Add the integers. Draw a picture of the tiles you used. Write the addition equation.
- Repeat the activity for a positive integer and a negative integer.
- Repeat the activity for two different negative integers.



Reflect & Share

Share your equations with another pair of classmates. How did you use the tiles to find a sum of integers? How can you predict the sign of the sum?

Connect

- To add two positive integers: $(+5) + (+4)$. We can model each integer with tiles.



Combine the tiles. There are 9 yellow tiles.

They model $+9$.

So, $(+5) + (+4) = +9$

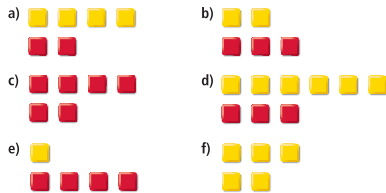
This is an addition equation.

Practice

Use coloured tiles.

1. What sum does each set of tiles model?

Write the addition equation.



2. What sum does each set of tiles model?

How do you know you are correct?

- 3 yellow tiles and 2 red tiles
- 3 yellow tiles and 4 red tiles
- 2 red tiles and 2 yellow tiles

3. Use coloured tiles to represent each sum. Find each sum.

Sketch the tiles you used. What do you notice?

- a) $(+2) + (-2)$
- b) $(-4) + (+4)$
- c) $(+5) + (-5)$

4. Add. Sketch coloured tiles to show how you did it.

- a) $(+2) + (+3)$
- b) $(-3) + (+4)$
- c) $(-4) + (-1)$
- d) $(+1) + (-1)$
- e) $(-3) + (-4)$
- f) $(+5) + (-2)$

5. Add. Write the addition equations.

- a) $(+4) + (+3)$
- b) $(-7) + (+5)$
- c) $(-4) + (-5)$
- d) $(+8) + (-1)$
- e) $(-10) + (-6)$
- f) $(+4) + (-13)$

6. Represent each sentence with integers, then find each sum.

- a) The temperature drops 3°C and rises 4°C .
- b) Marie earned $\$5$ and spent $\$3$.
- c) A stock rises 15c , then falls 7c .
- d) Jerome moves his game piece 3 squares backward, then 8 squares forward.
- e) Duma deposits $\$12$, then withdraws $\$5$.



- To add a negative integer and a positive integer: $(-6) + (+9)$. We can model each integer with tiles. Circle zero pairs.



There are 6 zero pairs.

There are 3 yellow tiles left.

They model $+3$.

So, $(-6) + (+9) = +3$

- To add two negative integers: $(-3) + (-7)$. We can model each integer with tiles.



Combine the tiles. There are 10 red tiles.

They model -10 .

So, $(-3) + (-7) = -10$

Example

The temperature rises 5°C , then falls 8°C .

- a) Represent the above sentence with integers. b) Find the overall change in temperature.

A Solution

a) $+5$ represents a rise of 5°C .

-8 represents a fall of 8°C .

Using integers, the sentence is: $(+5) + (-8)$

b) Model each integer with tiles.

Circle zero pairs.



There are 3 red tiles left.

They model -3 .

So, $(+5) + (-8) = -3$

The overall change in temperature is -3°C .

7. Use question 6 as a model.

Write 3 integer addition problems.

Trade problems with a classmate.

Solve your classmate's problems with coloured tiles.

8. Copy and complete.

a) $(+5) + \square = +8$

b) $\square + (-3) = -4$

c) $(+3) + \square = +1$

d) $(-5) + \square = -3$

e) $(+2) + \square = +1$

f) $\square + (-6) = 0$

9. **Assessment Focus**

a) Add: $(+3) + (-7)$

b) Suppose you add the integers in the opposite order:

$(-7) + (+3)$. Does the sum change?

Use coloured tile drawings and words to explain the result.

c) How is $(-3) + (+7)$ different from $(+3) + (-7)$? Explain.

d) Repeat parts a to c with a sum of integers of your choice.

What do you notice?

10. **Take It Further** Add. Sketch coloured tiles to show how you did it.

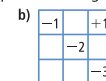
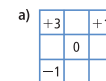
a) $(+1) + (+2) + (+3)$

b) $(+2) + (-1) + (+3)$

c) $(-3) + (-1) + (-1)$

d) $(+4) + (-3) + (+1)$

11. **Take It Further** In a magic square, every row, column, and diagonal has the same sum. Copy and complete each magic square. How did you do it?



12. **Take It Further** Copy each integer pattern.

What do you add each time to get the next term?

Write the next 4 terms.

a) $+8, +4, 0, -4, \dots$

b) $-12, -9, -6, -3, \dots$

Reflect

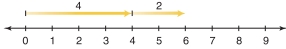
Talk to a partner. Tell how you used coloured tiles to add two integers when the integers have:

- the same signs
- opposite signs

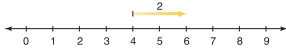
2.3 Adding Integers on a Number Line

Focus Add integers using number lines.

We can show the addition of whole numbers on a number line: $4 + 2 = 6$
Draw 2 arrows.



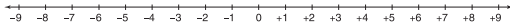
Or, begin at 4, and draw 1 arrow.



We can also show the addition of integers on a number line.

Explore

You will need copies of a number line.



- ▶ Choose two different positive integers. Use a number line to add them. Write the addition equation.
- ▶ Repeat the activity for a positive integer and a negative integer.
- ▶ Repeat the activity for two different negative integers.
- ▶ What happens when you add $+2$ and -2 ?

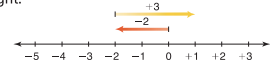


Reflect & Share

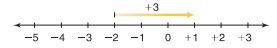
Compare your strategies for adding on a number line with those of your classmates. Use coloured tiles to check the sums. Why do you think integers such as $+2$ and -2 are called **opposite integers**?

Connect

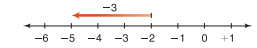
- ▶ To add a positive integer, move right (in the positive direction).
 $(-2) + (+3)$
Start at 0.
Draw an arrow 2 units long, pointing left.
This arrow represents -2 .
From -2 , draw an arrow 3 units long, pointing right.
This arrow represents $+3$.
The arrow head is at $+1$.
So, $(-2) + (+3) = +1$



Notice that the first arrow ends at the first integer. So, we could start at that integer, and use only 1 arrow to find the sum.

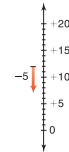


- ▶ To add a negative integer, move left (in the negative direction).
 $(-2) + (-3)$
Start at -2 .
Draw an arrow 3 units long, pointing left.
This arrow represents -3 .
The arrow head is at -5 .
So, $(-2) + (-3) = -5$



We can use the same method to add integers on a vertical number line.

- ▶ The temperature is 12°C . It falls 5°C .
Find the final temperature.
 $(+12) + (-5)$
Start at $+12$.
Draw an arrow 5 units long, pointing down.
This arrow represents -5 .
The arrow head is at $+7$.
So, $(+12) + (-5) = +7$
The final temperature is 7°C .



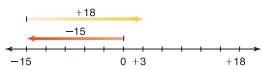
Example

Sandra and Joe buy and sell CDs at a flea market. One day in August, they bought 3 CDs for \$5 each. They sold 2 CDs for \$9 each.

- a) Write the expenses and income as integers.
- b) Did Sandra and Joe make money or lose money that day in August? Explain.

A Solution

- a) Expenses: $(-5) + (-5) + (-5) = -15$; they spent \$15.
Income: $(+9) + (+9) = +18$; they made \$18.
- b) Draw a number line.
Add expenses and income.



$$(-15) + (+18) = +3$$

Since the sum of the expenses and income is positive, Sandra and Joe made money. They made \$3.

Another Strategy

We could use coloured tiles.

Practice

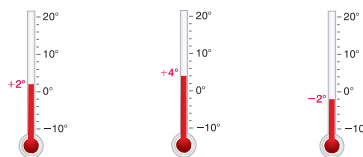
1. Use a number line to represent each sum.

a) $(+1) + (+3)$	b) $(-1) + (+3)$	c) $(-3) + (+1)$	d) $(-1) + (-3)$
e) $(-3) + (-4)$	f) $(-3) + (+4)$	g) $(+3) + (-4)$	h) $(+3) + (+4)$
2. Use a number line to add.

a) $(+4) + (+2)$	b) $(+5) + (-3)$	c) $(-4) + (-2)$	d) $(-8) + (+2)$
e) $(-6) + (-7)$	f) $(+1) + (-6)$	g) $(-5) + (+2)$	h) $(+8) + (+4)$
3. a) Reverse the order of the integers in question 2, then add.
b) Compare your answers to the answers in question 2. What do you notice?
c) Make a general statement about your observations.

4. Look at these thermometers. Find each temperature after:

a) it falls 4°C	b) it falls 7°C	c) it rises 6°C
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5. a) The temperature rises 7°C , then drops 2°C . What is the overall change in temperature?
b) Adrian loses \$4, then earns \$8. Did Adrian gain or lose overall?
c) The value of a stock went up \$3, then down \$2. What was the final change in the value of the stock?

6. Opposite integers are the same distance from 0 but are on opposite sides of 0.



- a) Write the opposite of each integer.

i) $+2$	ii) -5	iii) $+6$	iv) -8
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- b) Add each integer to its opposite in part a.
- c) What do you notice about the sum of two opposite integers?

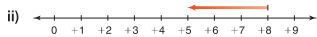
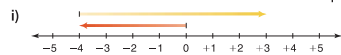
7. Use a number line. For each sentence below:
 - a) Write each number as an integer.
 - b) Write the addition equation.

- i) Explain your answer in words.
 - i) You take 5 steps backward, then 10 steps backward.
 - ii) You withdraw \$5, then deposit \$8.
 - iii) A deep sea diver descends 8 m, then ascends 6 m.
 - iv) A person drives a snowmobile 4 km east, then 7 km west.
 - v) A person gains 6 kg, then loses 10 kg.



Mid-Unit Review

8. a) Write the addition equation modelled by each number line.
b) Describe a situation that each number line could represent.



9. **Assessment Focus** Is each statement always true, sometimes true, or never true?

Use a number line to support your answers.

- a) The sum of two opposite integers is 0.
b) The sum of two positive integers is negative.
c) The sum of two negative integers is negative.
d) The sum of a negative integer and a positive integer is negative.
10. **Take It Further** Add.
a) $(+4) + (+3) + (-6)$ b) $(-2) + (-4) + (+1)$
c) $(-5) + (+3) + (-4)$ d) $(+6) + (-8) + (+2)$
11. **Take It Further** The temperature in Calgary, Alberta, was -2°C . A Chinook came through and the temperature rose 15°C . At nightfall, it fell 7°C . What was the final temperature? Support your answer with a drawing.



Reflect

Compare adding on a number line to adding with coloured tiles. Which method do you prefer? When might you need to use a different method?

LESSON

- 2.1 1. Use coloured tiles to model each integer in two different ways. Draw the tiles.
a) -5 b) 0
c) $+8$ d) -1
e) $+3$ f) -7
2. Suppose you have 8 red tiles. How many yellow tiles would you need to model $+3$? How do you know?
- 2.2 3. What sum does each set of tiles model? How do you know you are correct? Write the addition equations.
a) 6 yellow tiles and 1 red tile
b) 5 yellow tiles and 7 red tiles
c) 4 yellow tiles and 4 red tiles
4. Use coloured tiles to add. Draw pictures of the tiles you used.
a) $(+4) + (-1)$ b) $(-3) + (-2)$
c) $(-5) + (+1)$ d) $(+6) + (+3)$
e) $(-4) + (-8)$ f) $(+4) + (+8)$
- 2.3 5. Use a number line to add. Write the addition equations.
a) $(+3) + (+2)$ b) $(-5) + (-1)$
c) $(-10) + (+8)$ d) $(+6) + (-5)$
e) $(-8) + (+8)$ f) $(-5) + (+12)$
6. a) Add. $(+4) + (-5)$
b) Find 4 different pairs of integers that have the same sum as part a.
7. Write an addition equation for each situation.
a) Puja earned \$50, and spent \$20. How much did Puja then have?
b) The temperature is 5°C , then drops 10°C . What is the final temperature?
c) The population of a city was 124 000, then it dropped by 4000 people. What was the population then?
d) A plane was cruising at an altitude of 12 000 m, then dropped 1200 m. What was the cruising altitude then?
8. a) Write the addition equation modelled by each number line.
b) Describe a situation that each number line could represent.
- i)
-
- ii)
-
9. Each integer below is written as the sum of consecutive integers.
 $(+5) = (+2) + (+3)$
 $(+6) = (+1) + (+2) + (+3)$
Write each of these integers as the sum of consecutive integers.
a) $+10$ b) 0 c) $+2$
d) $+7$ e) $+4$ f) $+8$