## 2.4 Subtracting Integers with Tiles

Focus Use coloured tiles to subtract integers.

To add integers, we combine groups of tiles.
To subtract integers, we do the reverse:
we remove tiles from a group.

Recall that equal numbers of
red and yellow tiles model 0.
For example, $+5$ and -5 form 5 zero pairs,
and $(-5) + (+5) = 0$

Adding a zero pair to a set of tiles does not change its value.
For example, $(-3) + 0 = -3$
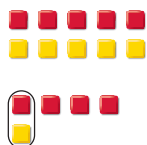
Explore

You will need coloured tiles.
Use tiles to subtract.
Add zero pairs when you need to.
Sketch the tiles you used in each case.
- $(+5) - (+3)$
- $(+5) - (-3)$
- $(-3) - (+5)$
- $(-3) - (-5)$

Reflect & Share

Compare your results with those of another pair of classmates.
Explain why you may have drawn different sets of tiles, yet both may be correct.
When you subtracted, how did you know how many tiles to use
to model each integer? How did adding zero pairs help you?

Connect

To use tiles to subtract integers, we model the first integer,
then take away the number of tiles indicated by the second integer.

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We can use tiles to subtract: $(+5) - (+9)$
Model $+5$.

There are not enough tiles to take away $+9$.
To take away $+9$, we need 4 more yellow tiles.

We add zero pairs without changing the value.
Add 4 yellow tiles and 4 red tiles. They represent 0.

By adding 0, the integer the tiles represent has not changed.
Now take away the 9 yellow tiles.

Since 4 red tiles remain, we write: $(+5) - (+9) = -4$

> This is a subtraction equation.

Example

Use tiles to subtract.
a) $(-2) - (-6)$ b) $(-6) - (+2)$ c) $(+2) - (-6)$

A Solution

a) $(-2) - (-6)$
   Model -2.
   There are not enough tiles to take away -6.
   To take away -6, we need 4 more red tiles.

   We add zero pairs without changing the value.
   Add 4 red tiles and 4 yellow tiles.

   Now take away 6 red tiles.

   Since 4 yellow tiles remain, we write: $(-2) - (-6) = +4$

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b) $(-6) - (+2)$
   Model -6.
   There are no yellow tiles to take.
   We need 2 yellow tiles to take away.

   We add zero pairs.
   Add 2 yellow tiles and 2 red tiles.

   Now take away 2 yellow tiles.

   Since 8 red tiles remain, we write: $(-6) - (+2) = -8$

c) $(+2) - (-6)$
   Model $+2$.
   There are no red tiles to take.
   We need 6 red tiles to take away.

   We add zero pairs.
   Add 6 red tiles and 6 yellow tiles.

   Now take away 6 red tiles.

   Since 8 yellow tiles remain, we write: $(+2) - (-6) = +8$

Notice the results in the *Example*, parts b and c.
When we reverse the order in which we subtract two integers,
the answer is the opposite integer.
$(-6) - (+2) = -8$
$(+2) - (-6) = +8$

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Practice

1. Use tiles to subtract. Draw pictures of the tiles you used.
   a) $(+7) - (+4)$ b) $(-2) - (-2)$ c) $(-9) - (-6)$
   d) $(+4) - (+2)$ e) $(-8) - (-1)$ f) $(+3) - (+3)$

2. Use tiles to subtract.
   a) $(-1) - (-4)$ b) $(+3) - (+8)$ c) $(-4) - (-11)$
   d) $(+7) - (+8)$ e) $(-4) - (-6)$ f) $(+1) - (+10)$

3. Subtract.
   a) $(-4) - (-1)$ b) $(+8) - (+3)$ c) $(-11) - (-4)$
   d) $(+8) - (+7)$ e) $(-6) - (-4)$ f) $(+10) - (+1)$

4. Subtract. Write the subtraction equations.
   a) $(+4) - (-7)$ b) $(-2) - (+8)$ c) $(-9) - (+5)$
   d) $(+6) - (-8)$ e) $(-3) - (+6)$ f) $(-5) - (+7)$

5. Subtract.
   a) $(+4) - (+5)$ b) $(-3) - (+5)$ c) $(-4) - (+3)$
   d) $(-1) - (-8)$ e) $(+8) - (-2)$ f) $(+4) - (-7)$

6. Use questions 1 to 5 as models.
   Write 3 integer subtraction questions.
   Trade questions with a classmate.
   Solve your classmate's questions.

7. a) Use coloured tiles to subtract each pair of integers.
      i) $(+3) - (+1)$ and $(+1) - (+3)$
      ii) $(-3) - (-2)$ and $(-2) - (-3)$
      iii) $(+4) - (-3)$ and $(-3) - (+4)$
   b) What do you notice about each pair of questions in part a?

8. $(+5) - (-2) = +7$
   Predict the value of $(-2) - (+5)$.
   Explain your prediction, then check it.

9. **Assessment Focus** Use integers.
   Write a subtraction question that would give each answer.
   How many questions can you write each time?
   a) $+2$ b) -3 c) $+5$ d) -6

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10. Which expression in each pair has the greater value?
Explain your reasoning.
a) i) $(+3) - (-1)$ **ii)** $(-3) - (+1)$
b) i) $(-4) - (-5)$ **ii)** $(+4) - (+5)$

11. Take It Further
a) Find two integers with a sum of -1 and a difference of $+5$.
b) Create and solve a similar integer question.

12. Take It Further Copy and complete.
a) $(+4) - \square = +3$
b) $(+3) - \square = -1$
c) $\square - (+1) = +4$

13. Take It Further Evaluate.
a) $(+4) + (+1) - (+3)$
b) $(+1) - (+2) - (-1)$
c) $(-3) - (+1) + (+4)$
d) $(-2) - (-4) + (-1)$
e) $(+2) - (+1) - (+4)$
f) $(+1) - (+2) + (+1)$

14. Take It Further Here is a magic square.
a) Subtract $+4$ from each entry.
Is it still a magic square? Why?
b) Subtract -1 from each entry.
Is it still a magic square? Why?

0	+5	−2
−1	+1	+3
+4	−3	+2

Reflect

Here are 4 types of subtraction questions:
- (negative integer) − (negative integer)
- (negative integer) − (positive integer)
- (positive integer) − (positive integer)
- (positive integer) − (negative integer)
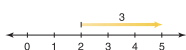Write a question for each type of subtraction.
Show how you use tiles to solve each question.

Recall how to model the subtraction of whole numbers with coloured tiles.
$7 - 5 = 2$

We can model this subtraction on a number line.
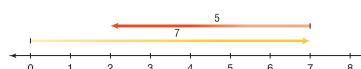
Subtraction is finding the difference. This number line shows how much more 7 is than 5.

Explore

You will need coloured tiles and copies of this number line.

$$-10 \ -9 \ -8 \ -7 \ -6 \ -5 \ -4 \ -3 \ -2 \ -1 \ 0 \ +1 \ +2 \ +3 \ +4 \ +5 \ +6 \ +7 \ +8 \ +9 \ +10$$

Step 1   Use tiles to subtract.
Sketch the tiles you used each time.
$(+7) - (+2)$ $(-7) - (-2)$
$(+7) - (-2)$ $(-7) - (+2)$

Step 2   Model each subtraction done with
tiles on a number line.

Step 3   Use any method. Add.
$(+7) + (-2)$ $(-7) + (+2)$
$(+7) + (+2)$ $(-7) + (-2)$

Step 4   Each expression in *Step 3* has a corresponding expression in *Step 1*.
What do you notice about the answers to corresponding expressions?
What patterns do you see in each subtraction and addition?
Check your pattern using other integers.

Reflect & Share

Compare your answers with those of another pair of classmates.
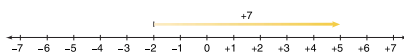How can you use addition to subtract two integers?

Connect

➤ To subtract two whole numbers, such as $5 - 2$,
we can think, "What do we add to 2 to get 5?"
We add 3 to 2 to get 5; so, $5 - 2 = 3$

We could also think: How much more is 5 than 2?

➤ We can do the same to subtract two integers.
For example, to subtract: $(+5) - (-2)$
Think: "What do we add to -2 to get $+5$?"

We add $+7$ to -2 to get $+5$; so, $(+5) - (-2) = +7$
We also know that $(+5) + (+2) = +7$.
We can look at other subtraction equations and
related addition equations.
$(+9) - (+4) = +5$ $(+9) + (-4) = +5$
$(-9) - (-4) = -5$ $(-9) + (+4) = -5$
$(-9) - (+4) = -13$ $(-9) + (-4) = -13$
$(+9) - (-4) = +13$ $(+9) + (+4) = +13$

In each case, the result of subtracting an integer is the
same as adding the opposite integer.
For example,
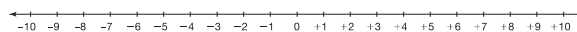$(-9) - (+4) = -13$ $(-9) + (-4) = -13$

Subtract $+4$.   Add -4.

➤ To subtract an integer, we add the opposite integer.
For example, to subtract: $(-3) - (-6)$
Add the opposite: $(-3) + (+6)$
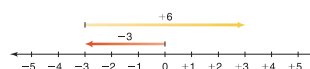
The opposite of -6 is $+6$.

So, $(-3) - (-6) = +3$

Example

Subtract.
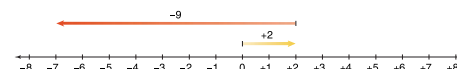**a)** $(+2) - (+9)$ **b)** $(-2) - (+9)$

A Solution

a) To subtract: $(+2) - (+9)$
Add the opposite: $(+2) + (-9)$
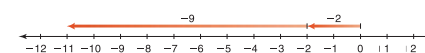Use a number line.
$(+2) + (-9) = -7$

Another Strategy
We could use coloured tiles.

b) To subtract: $(-2) - (+9)$
Add the opposite: $(-2) + (-9)$
Use a number line.
$(-2) + (-9) = -11$

Practice

1. Use a number line to subtract.
Use coloured tiles to check your answers.
a) $(+2) - (+1)$ **b)** $(+4) - (-3)$ **c)** $(-4) - (-1)$
d) $(-5) - (+2)$ **e)** $(-2) - (-6)$ **f)** $(-3) - (-7)$

2. a) Reverse the order of the integers in question 1, then subtract.
   b) How are the answers different from those in question 1? Explain.

3. Use a number line to subtract. Write the subtraction equations.
   a) $(+10) - (+5)$ **b)** $(+7) - (-3)$ **c)** $(-8) - (+6)$
   d) $(-10) - (+5)$ **e)** $(-4) - (+4)$ **f)** $(-4) - (-4)$

4. Rewrite using addition to find each difference.
   a) $(+6) - (+4)$ **b)** $(-5) - (+4)$ **c)** $(-2) - (-3)$
   d) $(+4) - (-2)$ **e)** $(+1) - (+1)$ **f)** $(+1) - (-1)$

5. What is the difference in temperatures?
   How can you subtract to find out?
   a) A temperature 7°C above zero and a temperature 5°C below zero
   b) A temperature 15°C below zero and a temperature 8°C below zero
   c) A temperature 4°C below zero and a temperature 9°C above zero

6. What is the difference in golf scores?
   How can you subtract to find out?
   a) A golf score of 2 over par and a golf score of 6 under par
   b) A golf score of 3 under par and a golf score of 8 under par
   c) A golf score of 5 under par and a golf score of 4 over par

7. a) The table shows the average afternoon
   temperatures in January and April for four
   Canadian cities.
   What is the rise in temperature from January
   to April for each city? Show your work.
   b) Which city has the greatest difference
   in temperatures?
   How do you know?

	City	January Temperature	April Temperature
i)	Calgary	−4°C	+13°C
ii)	Iqaluit	−22°C	−10°C
iii)	Toronto	−3°C	+12°C
iv)	Victoria	+7°C	+13°C

8. **Assessment Focus**
   a) Subtract: $(-6) - (+11)$
   b) Suppose we subtract the integers in the
      opposite order: $(+11) - (-6)$
      How does the answer compare with the
      answer in part a?
      Use number lines to explain.
   c) How is $(+6) - (-11)$ different from
      $(-6) - (+11)$? Explain.

9. Show three ways that $+4$ can be written as the
   difference of two integers.

10. **Take It Further** Use patterns to subtract.
   a) Subtract: $(+2) - (+5)$
      Start the pattern with $(+6) - (+5) = +1$.
   b) Subtract: $(+7) - (-3)$
      Start the pattern with $(+7) - (+4) = +3$.
   c) Subtract: $(-3) - (+7)$
      Start the pattern with $(+8) - (+7) = +1$.

11. **Take It Further** Copy each integer pattern.
   Write the next 4 terms.
   What is the pattern rule?
   a) $+6, +2, -2, \ldots$ **b)** $-3, -1, +1, \ldots$
   c) $+5, +12, +19, \ldots$ **d)** $+1, 0, -1, \ldots$

12. **Take It Further** Evaluate.
   a) $(+4) - (+2) - (+1)$ **b)** $(-2) - (+1) - (-4)$
   c) $(-1) + (-2) - (+1)$ **d)** $(+5) - (+1) + (-2)$
   e) $(+10) - (+3) - (-5)$ **f)** $(-7) - (+1) + (-3)$

Reflect

How is the subtraction of integers related
to the addition of integers?
Use coloured tiles or a number line to show your thinking.

Writing to Reflect on Your Understanding

As you work through a math unit, you will
come across many new ideas.

Sometimes it is hard to decide what you
already know.
What you know can often help you
understand the new ideas.

You can use a Homework Log to help you
reflect on your understanding.

Using a Homework Log

As you work through your homework, ask yourself:
• What is the key idea?
• How difficult is the homework for me?
• Which questions am I able to do?
• Which questions do I need help with?
• What questions could I ask to help me with my homework?

Tips for Writing a Homework Log

• Write so that someone else can understand you.
• Write out a question that you cannot solve.
• Describe 3 ways you tried to solve the question.
• Write a question you can ask to help you better
  understand your homework.
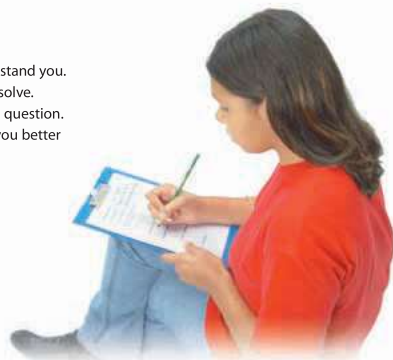
Reading and Writing in Math

Here is a sample Homework Log.

Name: _Asad_

Homework Log

The homework was . . . P26 #5−12

The key concept was . . . Subtracting integers on a number line

Overall, I'd rate the difficulty level of the homework as . . .
   Easy 0 1 2 3 4 5 6 7 8 9 10 Hard

One question I had difficulty with but solved was . . .
   What is the difference in temperatures?
   A temperature 7°C above zero and a temperature
   5°C below zero
      +7°C
      0°C }12°C
      −5°C

A question I couldn't solve was . . .
   Evaluate: $(+5) - (+1) + (-2)$

To solve it I tried these things . . .
   1. I used my calculator but I know I should be able
      to do it without one.
   2. I tried to model it on a number line but I didn't
      know which way to draw the arrows.
   3. I looked at the example in the book. It says to
      "add the opposite" but I don't know what that means.

Questions for experts . . .
   What does "add the opposite" mean?
   How do you take away a negative integer?

Check

• Complete a Homework Log for your next homework assignment.
• Share your Homework Log with a classmate.
• Try to help each other with questions that you were unable to solve.
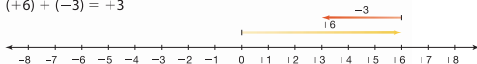
Unit Review

What Do I Need to Know?

✅ **Adding Integers**

- You can use tiles to add integers.
  $(-7) + (+2) = -5$

- You can use a number line to add integers.
  $(+6) + (-3) = +3$

✅ **Subtracting Integers**

- You can use tiles to subtract integers: $(+3) - (-7)$
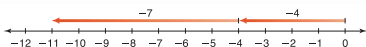 We need enough red tiles to take away 7 of them.
  Model $+3$:

  Since there are not enough tiles to take away -7, add 7 yellow tiles and 7 red tiles. Now take away 7 red tiles. There are 10 yellow tiles left.

  $(+3) - (-7) = +10$

- You can also subtract by adding the opposite:
  $(-5) - (-8) = (-5) + (+8)$
  $= +3$
- You can use a number line to subtract integers.
  $(-4) - (+7)$
  Add the opposite: $(-4) + (-7)$
  Use a number line.
  $(-4) - (+7) = -11$

LESSON

2.1 **1.** Suppose you have 17 red tiles. How many yellow tiles would you need to model:
a) -12?   b) 0?
c) $+20$?   d) -17?
How do you know?

2. Write the integer suggested by each of the following situations. Draw yellow or red tiles to model each integer. Explain your choice.
a) The temperature rises 8℃.
b) The price of 1 L of gas falls 5¢.
c) You deposit $12 in your bank account.
d) You take 7 steps backward.
e) The time is 9 s before take-off.

2.2 **3.** What sum does each set of tiles model?
a) 5 red tiles and 2 yellow tiles
b) 6 yellow tiles and 5 red tiles
c) 6 yellow tiles and 7 red tiles
d) 8 yellow tiles and 8 red tiles

4. Represent each sentence with integers, then find each sum.
a) The temperature was -6℃, then rose 4℃.
b) Surinder withdrew $25, then deposited $13.
c) A stock gained $15, then lost $23.
d) A submarine was 250 m below sea level, then ascended 80 m.

5. a) Find 4 pairs of integers that have the sum -5.
b) Find 4 pairs of integers that have the sum $+4$.

2.3 **6.** The temperature at 6 a.m. is -10℃. During the day, the temperature rises 17℃. What is the new temperature? Write an addition equation to represent this situation. Use a vertical number line to support your answer.

7. a) Write an addition equation modelled by each number line.
b) Describe a situation that each number line could represent.
i)

ii)
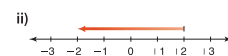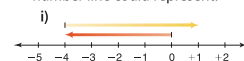
2.2 **8.** Use tiles to add or subtract.
2.4 a) $(-1) + (+3)$
b) $(+3) + (-4)$
c) $(-2) - (+3)$
d) $(-1) - (-3)$

LESSON

2.3 **9.** Use a number line to add or subtract.
2.5 a) $(-1) + (+3)$ b) $(+6) + (-4)$
c) $(-4) - (+6)$ d) $(-5) - (-3)$

10. When you add two positive integers, their sum is always a positive integer.
When you subtract two positive integers, is their difference always a positive integer? Explain.

11. a) What temperature is 7℃ warmer than 2℃?
b) What temperature is 5℃ warmer than -5℃?
c) What temperature is 8℃ cooler than 2℃?
d) What temperature is 4℃ cooler than -3℃?

2.4 **12.** Use tiles or a number line to subtract.
2.5 Write the subtraction equations.
a) $(+4) - (+1)$ b) $(+5) - (-1)$
c) $(+2) - (-2)$ d) $(-4) - (+1)$
e) $(-6) - (-2)$ f) $(-10) - (-5)$
g) $(-4) - (-2)$ h) $(-5) - (-10)$

13. Subtract.
a) $(+7) - (+2)$ b) $(-7) - (+3)$
c) $(-4) - (-5)$ d) $(+3) - (+3)$
e) $(+3) - (-3)$ f) $(-3) - (-2)$

14. Use tiles or a number line. Find the difference between:
a) a temperature of $+5$℃ and -7℃
b) an elevation of -100 m and $+50$ m

15. What is the difference in heights? How can you subtract to find out?
a) A water level of 2 m below sea level and a water level of 7 m above sea level
b) A balloon 25 m above ground and a balloon 11 m above ground

16. What is the difference in masses? How can you subtract to find out?
a) A gain of 9 kg and a loss of 3 kg
b) A loss of 6 kg and a loss of 5 kg

17. We measure time in hours. Suppose 12 noon is represented by the integer 0.
a) Which integer represents 1 p.m. the same day?
b) Which integer represents 10 a.m. the same day?
c) Find the difference between these times in 2 ways. Show your work.

18. a) Find 5 pairs of integers with a difference of $+6$.
b) Find 5 pairs of integers with a difference of -3.