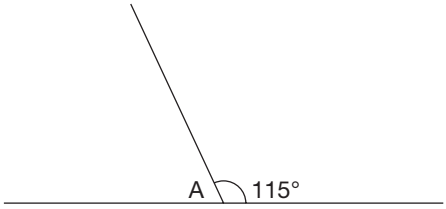
LESSON 6

Investigating Angles in a Triangle

Without using a protractor, what is the measure of angle A?
How do you know?

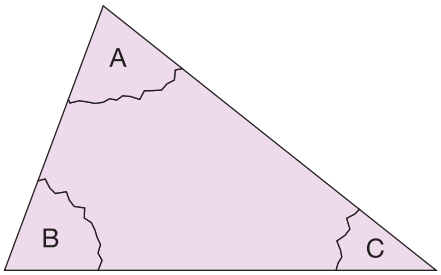


Explore

You will need a ruler, scissors, and a protractor.

➤ Draw a triangle to match each description below:
- a triangle with one right angle
- a triangle with one obtuse angle
- a triangle with all acute angles

Use a protractor to measure the angles in each triangle.
Record the measures in a table.



➤ Cut out one of the triangles. Cut off its angles.
Place the vertices of the three angles together so adjacent sides touch. What do you notice?

➤ Repeat the activity with the other two triangles.
What can you say about the sum of the angles in each triangle?

➤ Use the measures in your table.
Find the sum of the angles in each triangle.
Does this confirm your results from cutting off the angles?
Explain.

Show *and* Share

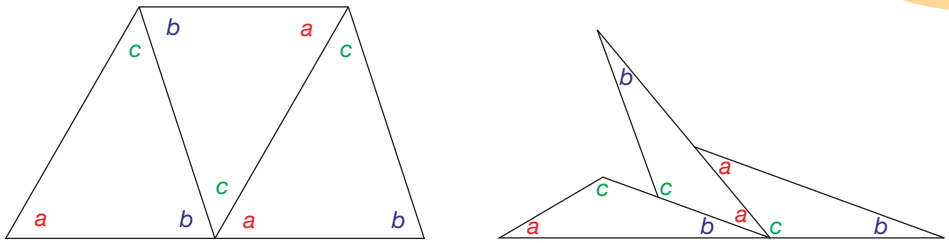
Compare your results with those of another pair of classmates.
What can you say about the sum of angles in a triangle?
Do you think this would be true for all triangles?
Explain your thinking.

Connect

➤ We can show that the sum of the **interior angles** in a triangle is the same for any triangle.

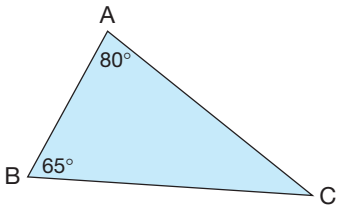
An interior angle is an angle inside a triangle or other polygon.

Arrange 3 congruent triangles as shown.



The arrangements show that angles *a*, *b*, and *c* make a straight angle.
So, $a + b + c = 180°$
The sum of the angles in a triangle is 180°.

➤ We can use the sum of the angles in a triangle to find the measure of the third angle in this triangle.



We often refer to an angle using the letter of its vertex. For example, the 80° angle in triangle ABC is ∠A.

The sum of the angles in a triangle is 180°.
So, $\angle A + \angle B + \angle C = 180°$
Since $\angle A = 80°$ and $\angle B = 65°$,
$80° + 65° + \angle C = 180°$ Add the angles.
$145° + \angle C = 180°$

Solve the equation by inspection.
Which number do we add to 145 to get 180?
The measure of ∠C is 35°.

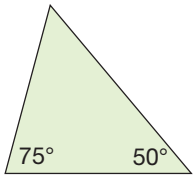
To check, we can find the sum of the 3 angles:
$\angle A + \angle B + \angle C = 80° + 65° + 35°$
$= 180°$
So, the answer is correct.

Practice

1. Draw 3 different triangles on dot paper. Measure and record each angle.
   Find the sum of the measures of the angles for each triangle.

2. Determine the measure of the third angle without measuring.

   a)



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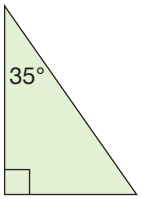

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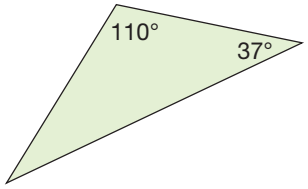
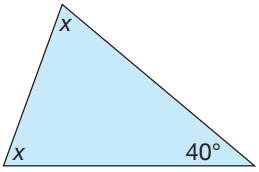


3. The two unknown angles in each triangle below are equal.
   Determine the measure of each unknown angle without measuring.
   Explain the strategy you used.

   a)



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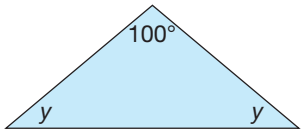

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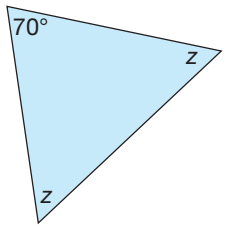


4. Two angles of a triangle are given.
   Find the measure of the third angle.
   a) 55°, 105° **b)** 45°, 90°
   c) 30°, 60° **d)** 25°, 125°

5. Vegreville, Alberta, is home to the world's largest known Ukrainian egg. It has 1108 triangular pieces with three angles of equal measure.
   Find the measure of each angle.
   Explain your strategy.

6. Is it possible for a triangle to have:
   a) more than 1 obtuse angle?
   b) 2 right angles?
   c) 3 acute angles?
   Explain your thinking.
   Use pictures and words.

7. Find the measure of the third angle in each triangle described below. Then, draw the triangle.
Explain how you found each measure.

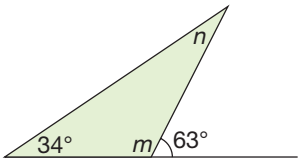
a) A triangle with two angles measuring 65° and 55°

b) A triangle with two equal angles; each measures 40°

c) A right triangle with a 70° angle

8. Find the measures of the angles labelled *m* and *n*.
Explain the strategy you used.

a)



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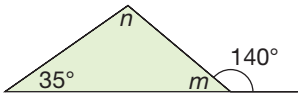

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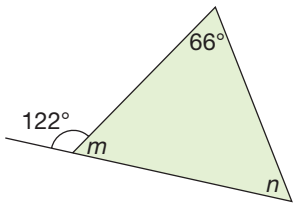
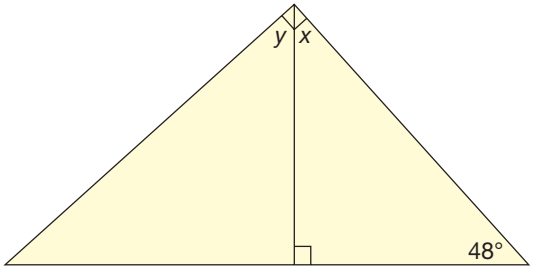


9. Find the measures of the angles labelled *x* and *y*.
Show your work. Explain the strategy you used.

a)



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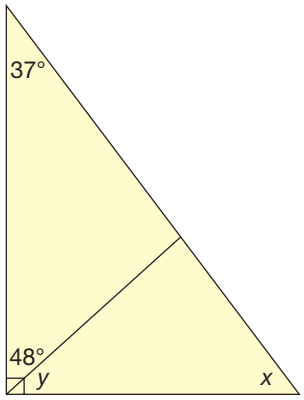
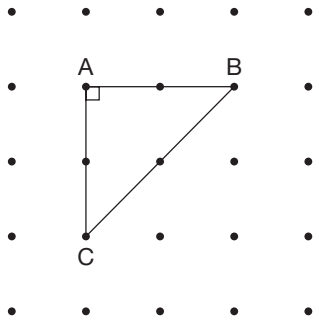


10. Use a geoboard and geobands or square dot paper.
Construct △ABC.



a) Find the unknown angle measures.
Check your answers by measuring with a protractor.

b) Extend AB 1 unit right to D.
Extend AC 1 unit down to E. Join DE.

c) Predict the measure of each angle in the new triangle.
Use a protractor to check. Record your work.

d) Repeat steps b and c two more times.

e) What do you notice about all the triangles you created? Explain.

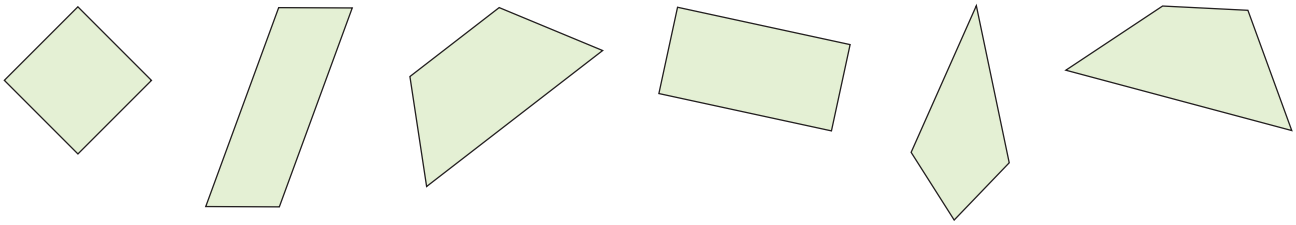
Reflect

Suppose your classmate missed today's lesson.
Explain how you know the sum of the angles in any triangle.

LESSON 7

Investigating Angles in a Quadrilateral

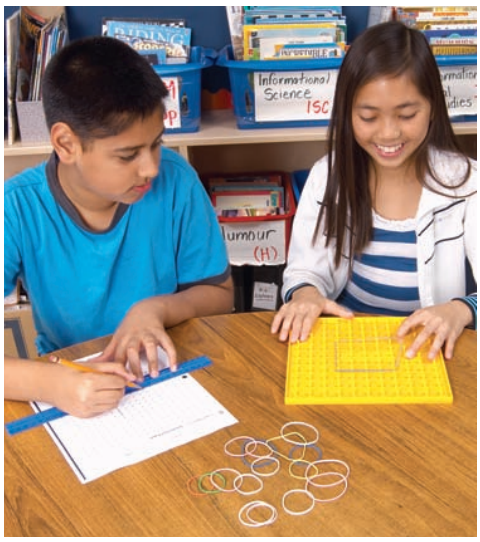
How are these quadrilaterals alike?
How are they different?



Explore

You will need a geoboard, geobands, a ruler, and square dot paper.
Draw each shape you make on dot paper.

- Make a square.
  What do you know about each interior angle?
  What is the sum of the angles in a square?
- Make a rectangle.
  What do you know about each interior angle?
  What is the sum of the angles in a rectangle?
- Make 2 different quadrilaterals.
  None of the angles can be right angles.
  Suppose you don't have a protractor.
  How can you find the sum of the angles in each quadrilateral?
- What can you say about the sum of the angles in a quadrilateral? Explain.

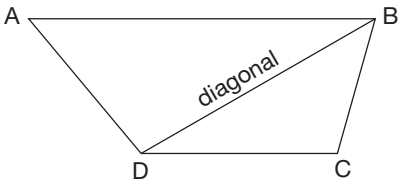


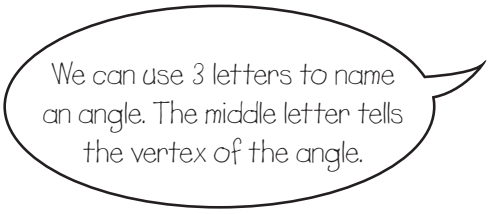
Show *and* Share

Compare your results with those of another pair of students.
How can you use what you know about triangles to find the sum of the angles in a quadrilateral?
Do you think this is true for all quadrilaterals? Why or why not?

Connect

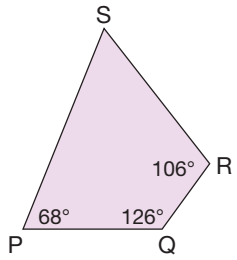
➤ The sum of the interior angles in a quadrilateral is the same for any quadrilateral.
A **diagonal** divides any quadrilateral into 2 triangles.





The sum of the angles in each triangle formed is 180°.
In △ABD, ∠ABD + ∠BDA + ∠DAB = 180°
In △DBC, ∠DBC + ∠BCD + ∠CDB = 180°
So, the sum of the angles in quadrilateral ABCD is 2 × 180° = 360°.

➤ We can use the sum of the angles in a quadrilateral to find the measure of ∠S in quadrilateral PQRS.



The sum of the angles in a quadrilateral is 360°.
So, ∠P + ∠Q + ∠R + ∠S = 360°
Since ∠P = 68°, ∠Q = 126°, and ∠R = 106°,
68° + 126° + 106° + ∠S = 360° Add the angles.
300° + ∠S = 360°

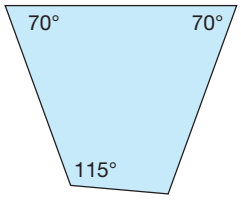
Solve the equation by inspection.
Which number do we add to 300 to get 360?
The measure of ∠S is 60°.

Practice

1. Draw 3 different quadrilaterals on dot paper.
Measure and record each angle.
Find the sum of the measures of the angles for each quadrilateral.

2. Find the unknown angle measure in each quadrilateral.

a)



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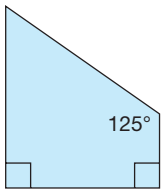

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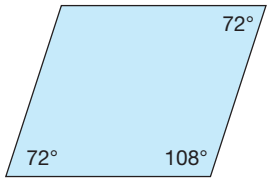

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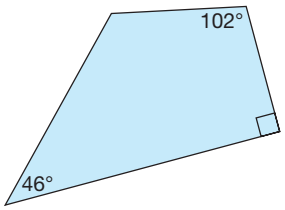

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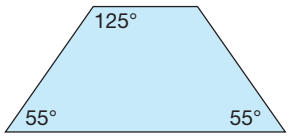

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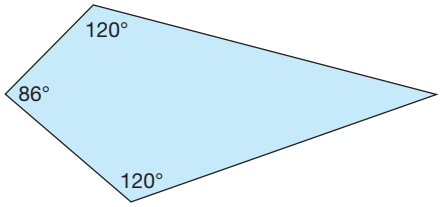


3. A student drew 4 different quadrilaterals.
She recorded the angle measures in a table.

Quadrilateral	∠A	∠B	∠C	∠D
a)	225°	36°	47°	42°
b)	81°	99°	81°	99°
c)	90°	45°	120°	105°
d)	123°	66°	108°	73°

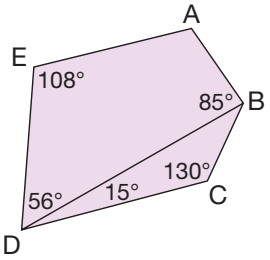
Did the student measure the angles in each quadrilateral correctly?
How do you know?

4. Use a geoboard and geobands and/or dot paper.
Try to make each quadrilateral below.
If you can make the quadrilateral, record your work on dot paper.
If you cannot make the quadrilateral, use what you know about the sum of the angles in a quadrilateral to explain why.

a) a quadrilateral with 4 right angles
b) a quadrilateral with 2 acute angles and 2 obtuse angles
c) a quadrilateral with only one right angle
d) a quadrilateral with 4 acute angles
e) a quadrilateral with 4 obtuse angles

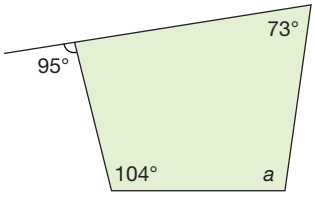
5. Look at this pentagon.

a) Find the measure of ∠A.
b) Find the measure of ∠DBC.
Show your work. Explain your thinking.



6. Find the measure of the angles labelled *a*, *b*, and *c*.
Show your work.

a)



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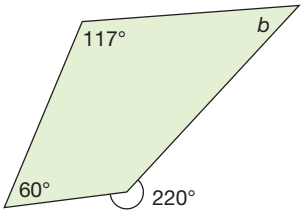

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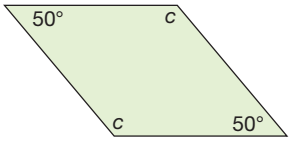
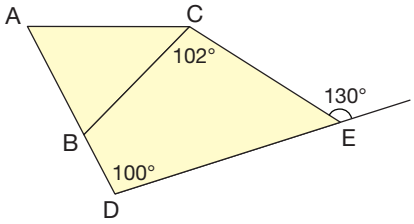
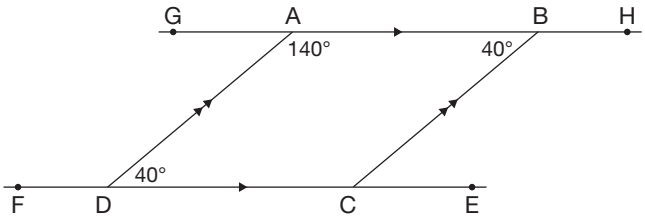


7. Find the measure of ∠ABC.
Show all the steps you took to find its measure.



8. Draw a rectangle. Draw its diagonals.
Measure one of the angles formed where the diagonals intersect.
Without using a protractor, find the measures of the other 3 angles.
Explain your strategy.
Repeat for 2 different quadrilaterals.
What do you notice?

9. Look at parallelogram ABCD.



a) Without using a protractor, find the measure of ∠BCD.
b) Find the measure of ∠BCE, ∠CBH, ∠ADF, and ∠DAG.
What strategy did you use?
c) List pairs of angles that have the same measure.
d) List pairs of angles that add to 180°.

Reflect

How did you use what you know about the sum of the angles in a triangle in this lesson?