Problem of the Week Problem B and Solution Alternate Dimensions

Problem

The four shapes to the right are each drawn with a horizontal base and a vertical height. Figure A is a right-angled triangle, Figure B is an isosceles triangle, Figure C is a square, and Figure D is a rectangle. The figures are not drawn to scale.



Using the following clues, determine the measure of the (horizontal) base and the measure of the (vertical) height of each figure.

- 1. The measure of the base of Figure A is the same as the measure of the base of Figure D.
- 2. The measure of the base of Figure A is one unit less than the measure of the base of Figure B.
- 3. The side length of Figure C is the same as the measure of the base of Figure A.
- 4. The measure of the height of Figure B is the same as the measure of the height of Figure A and also the same as the measure of the base of Figure B.
- 5. The area of Figure C is 9 square units.
- 6. The total area of all four figures is 38 square units.

Solution

From 5., since C is a square with an area of 9 square units, its side length is 3 units.

From 3., A must have a base that is 3 units long.

From 1., D has a base that is 3 units long as well.

From 2., the measure of the base of B is 1 unit greater than that of A, so B has a base that is 4 units long.

From 4., the heights of ${\sf A}$ and ${\sf B}$ thus both have a measure of 4 units.

We can now calculate that the area of triangle A is $\frac{1}{2} \times 3 \times 4 = 6$ square units and the area of triangle B is $\frac{1}{2} \times 4 \times 4 = 8$ square units. We also know that the area of square C is 9 square units. Summing, the total area of figures A, B, and C is 6 + 8 + 9 = 23 square units.

From 6., the total area of all figures is 38 square units, so the area of D is 38 - 23 = 15 square units. Since D is a rectangle with a base of measure 3, it thus has a height of measure $15 \div 3 = 5$ units.

Therefore,

Figure A has a base of measure 3 units and a height of measure 4 units;

Figure B has a base of measure 4 units and a height of measure 4 units;

Figure C is 3 units by 3 units; and

Figure D has a base of measure 3 units and a height of measure 5 units.

