Problem of the Week
Problem B and Solution
More Practice

Problem

When Nick shoots a basketball, he either sinks the shot or misses. For each shot Nick sinks, he is given 5 points by his father. For each missed shot, Nick's Dad takes 2 points away.

Nick attempts a total of 28 shots and ends up with zero points (i.e., he breaks even). How many shots did Nick sink?

Solution

Solution 1

Shots Sunk	Points Gained	Shots Missed	Points Lost	Points Gained − Points Lost
20	100	8	16	$100 - 16 = 84$
16	80	12	24	$80 - 24 = 56$
14	70	14	28	$70 - 28 = 42$
12	60	16	32	$60 - 32 = 28$
10	50	18	36	$50 - 36 = 14$
8	40	20	40	$40 - 40 = 0$

By trying different combinations of shots sunk and shots missed, as shown in the table, we see that Nick breaks even if he sinks 8 shots and misses 20 shots.

Solution 2

We make a couple of observations. First, to break even, Nick must have sunk an even number of shots. If he sunk an odd number of shots, he would have earned an odd number of points, since multiplying any odd number by 5 produces an odd number. But he always loses an even number of points for missed shots since multiplying any number by 2 produces an even number. An odd number minus an even number will never equal zero. Thus, Nick could not have broken even by sinking an odd number of shots.

Second, Nick sunk less than 14 shots (half of the 28 shots). If he sunk 14 shots, Nick would have gained $14 \times 5 = 70$ points, but only lost $2 \times 14 = 28$ points. He would not have broken even.

We have quickly reduced the number of possibilities for successful shots to $\{0, 2, 4, 6, 8, 10, 12\}$.

It does not take much time to narrow down to the correct solution (shown above). That is, Nick broke even by sinking 8 shots and missing 20 shots.

An algebraic solution is possible and is presented on the next page.

Solution 3

Let a represent the number of shots sunk and b represent the number of shots missed.

Then $5 \times a$ represents the number of points Nick gained for successful shots and $2 \times b$ represents the number of points Nick lost for missed shots

We want Nick to break even, so $5 \times a = 2 \times b$. This is generally written $5a = 2b$.

Since the total number of shots made was 28, $a + b = 28$.

By dividing both sides of the first equation by 5, we obtain $a = \frac{2}{5}b = 0.4b$.

Substituting $0.4b$ for a in the second equation we get

$$
\begin{aligned}
0.4b + b &= 28 \\
1.4b &= 28 \\
b &= \frac{28}{1.4} \\
b &= \frac{280}{14} \\
b &= 20
\end{aligned}
$$

Since b represents the number of missed shots, Nick missed 20 shots. Since Nick attempted a total of 28 shots and missed 20 shots, he sunk $28 - 20 = 8$ shots.

Therefore, Nick sunk 8 shots and missed 20 shots.

Equation solving techniques are generally covered in grade 7 or higher. This solution is provided just for information of those who may wish to get a glimpse of what is coming in future Mathematics courses.