Problem of the Week
Problem C and Solution
Sum of Everything

Problem

If you were to list the integers from 1 to 12, you would get the list $1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12$.
If you were to sum the digits of the integers in this list, you would get the sum
$1+2+3+4+5+6+7+8+9+(1+0)+(1+1)+(1+2) = 51$.

To the right are the integers from 1 to 100. Can you find the sum of all of the digits of these numbers?

1	2	3	4	5	6	7	8	9	10
11	12	13	14	15	16	17	18	19	20
21	22	23	24	25	26	27	28	29	30
31	32	33	34	35	36	37	38	39	40
41	42	43	44	45	46	47	48	49	50
51	52	53	54	55	56	57	58	59	60
61	62	63	64	65	66	67	68	69	70
71	72	73	74	75	76	77	78	79	80
81	82	83	84	85	86	87	88	89	90
91	92	93	94	95	96	97	98	99	100

Solution

(1) In the table above, each of the ten columns has a units digit that occurs ten times. So the sum of ALL of the units digits is

$$\begin{aligned} &10(1)+10(2)+10(3)+10(4)+10(5)+10(6)+10(7)+10(8)+10(9)+10(0) \\ &= 10(1+2+3+4+5+6+7+8+9+0) \\ &= 10(45) \\ &= 450 \end{aligned}$$

(2) Each of the ten columns has a tens digit from 0 to 9. So the sum of ALL of the tens digits is

$$\begin{aligned} &10(0+1+2+3+4+5+6+7+8+9) \\ &= 10(45) \\ &= 450 \end{aligned}$$

(3) The number 100 is the only number with a hundreds digit. We need to add 1 to our final sum.

(4) Now we add our results from (1), (2), and (3) to obtain the required sum.

$$\begin{aligned} \text{Sum of digits} &= \text{Units digit sum} + \text{Tens digit sum} + \text{Hundreds Digit} \\ &= 450 + 450 + 1 \\ &= 901 \end{aligned}$$

Therefore, the sum of all of the digits of the numbers from 1 to 100 is 901.